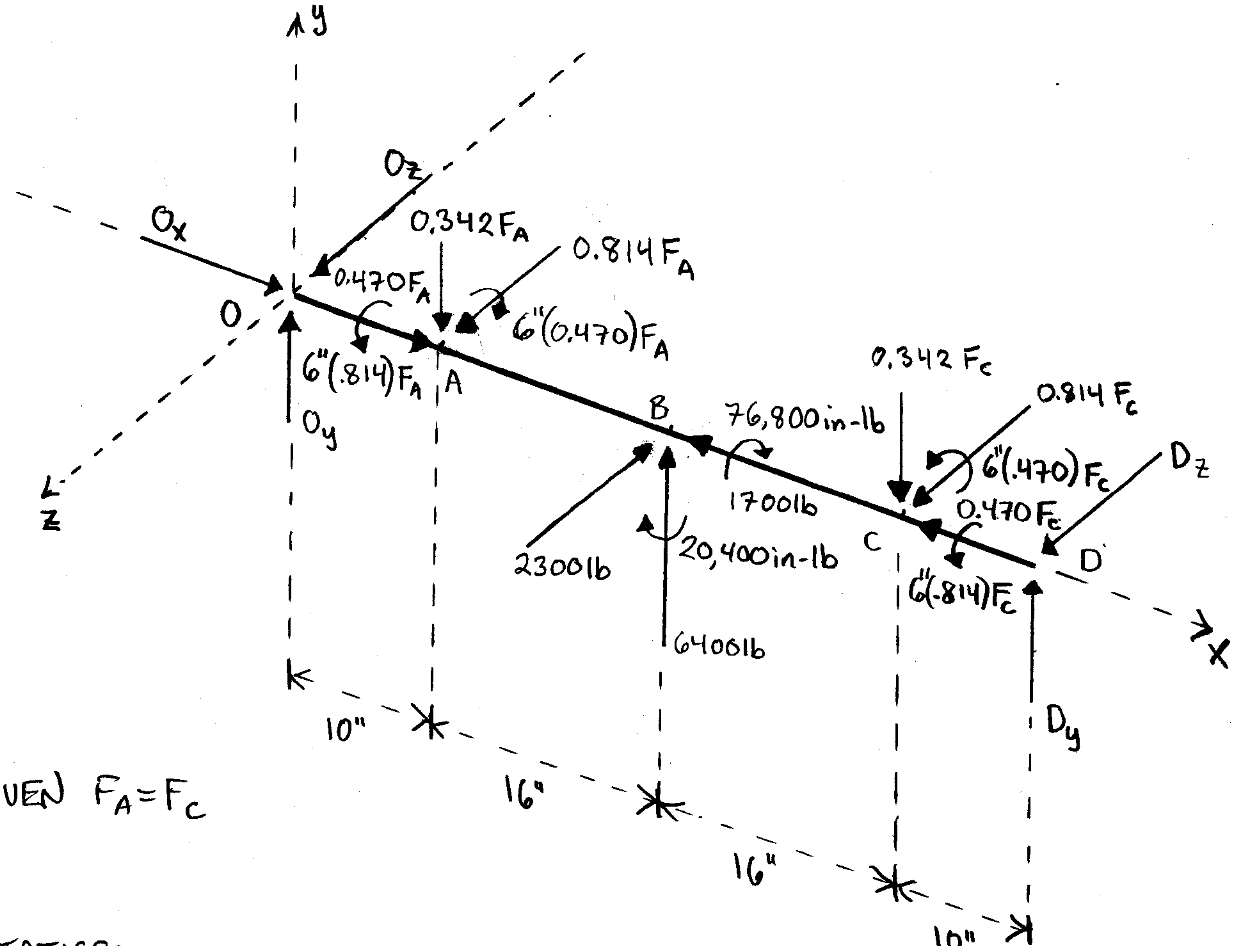
LAB #3 - FATIGUE

FBD OF SHAFT:



GIVEN $F_A = F_C$

STATICS:

$$\Sigma F_x = 0;\quad O_x + \cancel{0.470F_A} - 1700\text{lb} - \cancel{0.470F_c} = 0$$

$$\Sigma F_y = 0;\quad O_y - 0.342F_A + 6400\text{lb} - 0.342F_c + D_y = 0$$

$$\Sigma F_z = 0;\quad O_z + 0.814F_A - 2300\text{lb} + 0.814F_c + D_z = 0$$

$$\Sigma M_x^O = 0;\quad 6''(0.814)F_A - 76{,}800\text{ in-lb} + 6''(0.814)F_c = 0$$

$$\Sigma M_y^O = 0;\quad -10''(0.814)F_A + 26''(2300\text{lb}) - 20{,}400\text{in-lb} - 42''(0.814)F_c - 52''D_z = 0$$

$$\Sigma M_z^O = 0;\quad -52''(O_y) - \cancel{6''(0.470)}F_A + 42''(0.342)F_A - 26''(6400\text{lb}) + \cancel{6''(0.470)}F_c + 10''(0.342)F_c = 0$$

SOLVE THE ABOVE SIX EQUATIONS FOR THE FOLLOWING SIX UNKNOWNS: O_x, O_y, O_z, D_y, D_z & $F_A(=F_c)$ . . . SEE NEXT PAGE!

LAB #3 - FATIGUE (CONT'D)

ME 340
1/21/99

FROM $\Sigma M_x = 0 \rightarrow F_A = 76{,}800\text{ in-lb} / [2(6")0.814]$

$F_A = 7862.4\text{ lb} = F_c$

FROM $\Sigma F_x = 0 \rightarrow O_x = \cancel{-0.470(7862.4\text{ lb})} + 1700\text{ lb} + \cancel{0.470(7862.4\text{ lb})}$

$O_x = 1700\text{ lb}$

FROM $\Sigma M_y^o = 0 \rightarrow D_z = [(-10"-42")0.814(7862.4\text{ lb}) + 26"(2300\text{ lb}) - 20{,}400\text{ in-lb}]/52"$

$D_z = -5642.3\text{ lb}$

FROM $\Sigma M_z^D = 0 \rightarrow O_y = [(42"+10")0.342(7862.4\text{ lb}) - 26"(6400\text{ lb})]/52"$

$O_y = -511.06\text{ lb}$

FROM $\Sigma F_y = 0 \rightarrow D_y = 511.06\text{ lb} + 2(0.342)7862.4\text{ lb} - 6400\text{ lb}$

$D_y = -511.06\text{ lb lb}$

FROM $\Sigma F_z = 0 \rightarrow O_z = 2(-0.814)7862.4\text{ lb} + 2300\text{ lb} + 5642.3\text{ lb}$

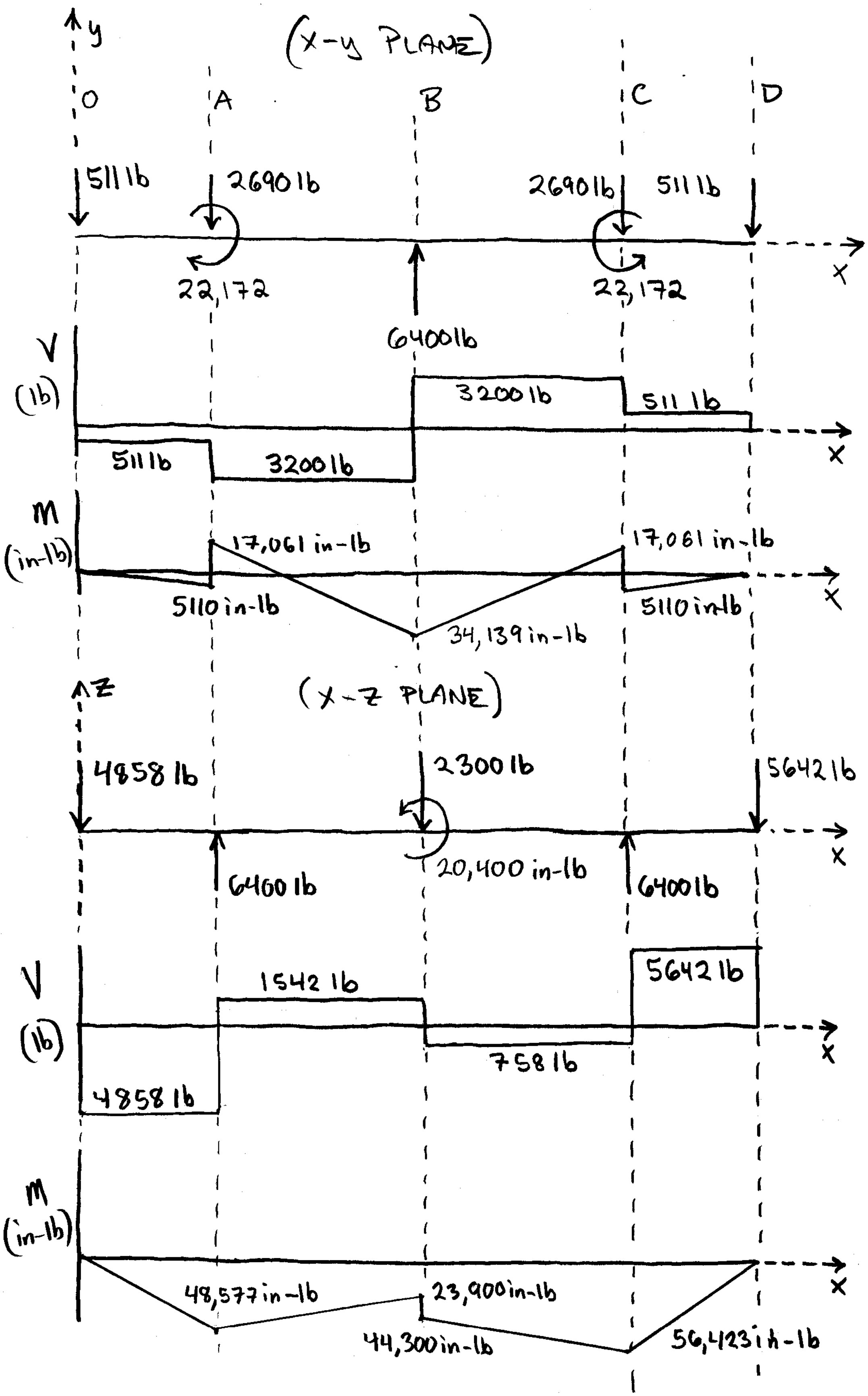
$O_z = -4857.7\text{ lb}$

SEE NEXT PAGE FOR SHEAR & MOMENT DIAGRAMS

REMEMBER $\frac{dM}{dx} = V$ & SIGN CONVENTION $\boxed{+}$

LAB #3 - FATIGUE (CONT'D)

SHEAR & MOMENT DIAGRAMS:



CRITICAL POINT IDENTIFICATION: (SHAFT IS UNIFORM IN DIAMETER, THIS POINT OF MAX STRESS WILL BE POINT OF MAX LOAD)

NEGLECT AXIAL & TRANSVERSE SHEAR FOR NOW. TORSION IS PRESENT AND CONSTANT IN MAGNITUDE BETWEEN POINTS A & C, THUS THE MAX LOADING WILL BE AT THE POINT OF LARGEST BENDING MOMENT, I.E., JUST TO THE LEFT OF POINT C, WHERE $M_{TOT} = \sqrt{(17{,}061)^2 + (-56{,}423)^2} = 58{,}946$ in-lb. BENDING & TORSION ARE MAX AT THIS POINT.

NOW WE SORT THE STRESSES INTO MEAN AND ALTERNATING LOADS. (FOR THE CRITICAL POINT)
THE TWO LOADS WE ARE CONSIDERING ARE THE BENDING AND TORSIONAL LOADS.

MEAN LOADS/STRESSES:

TORSION (ALWAYS PRESENT) $\tau = \frac{Tr}{J} K_{ts} = \frac{16T}{\pi d^3} K_{ts}$

$$\tau_{xy} = \frac{16(38{,}400 \text{ in-lb})}{\pi d^3}(3.1) = \frac{606{,}266 \text{ in-lb}}{d^3}$$

ALTERNATING LOADS/STRESSES:

BENDING (ALWAYS PRESENT, BUT ROTATION CAUSES ALTERNATION)

$$\sigma_x = \frac{Mc}{I} K_{fB} = \frac{32M}{\pi d^3} K_{fB} = \frac{32(58{,}946 \text{ in-lb})}{\pi d^3}(2.5) = \frac{1{,}501{,}000}{d^3} \text{ in-lb}$$

$$K_{fB} = 1 + q(K_{tB} - 1) = 1 + 0.82(2.8 - 1) = 2.476 \sim 2.5 \quad (q \text{ FROM FIG 6-36})$$

NOW WE NEED VON MISES EQUIVALENTS:

$$\sigma_m' = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}} = \sqrt{3}\,\frac{606{,}266}{d^3} = \frac{1{,}050{,}100}{d^3}$$

$$\sigma_a' = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}} = \sigma_x = \frac{1{,}501{,}000}{d^3} \text{ in-lb}$$

NEXT WE FIND THE MATERIAL'S ENDURANCE LIMIT

$$S_e' = \tfrac{1}{2} S_{ut}$$

$$S_e = \tfrac{1}{2} S_{ut} C_{LOAD} C_{SIZE} C_{SURF} C_{TEMP} C_{RELIAB}$$

$C_{LOAD} = 1$ (BENDING, EQN 6.7a NORTON)

$C_{SIZE} = 0.869 d^{-0.097} = 0.869(4)^{-0.097} = 0.76$ (EQN 6.7b)

(GUESS 4" DIA, VERIFY LATER)

$C_{SURF} = 0.76$ (FIGURE 6-26)

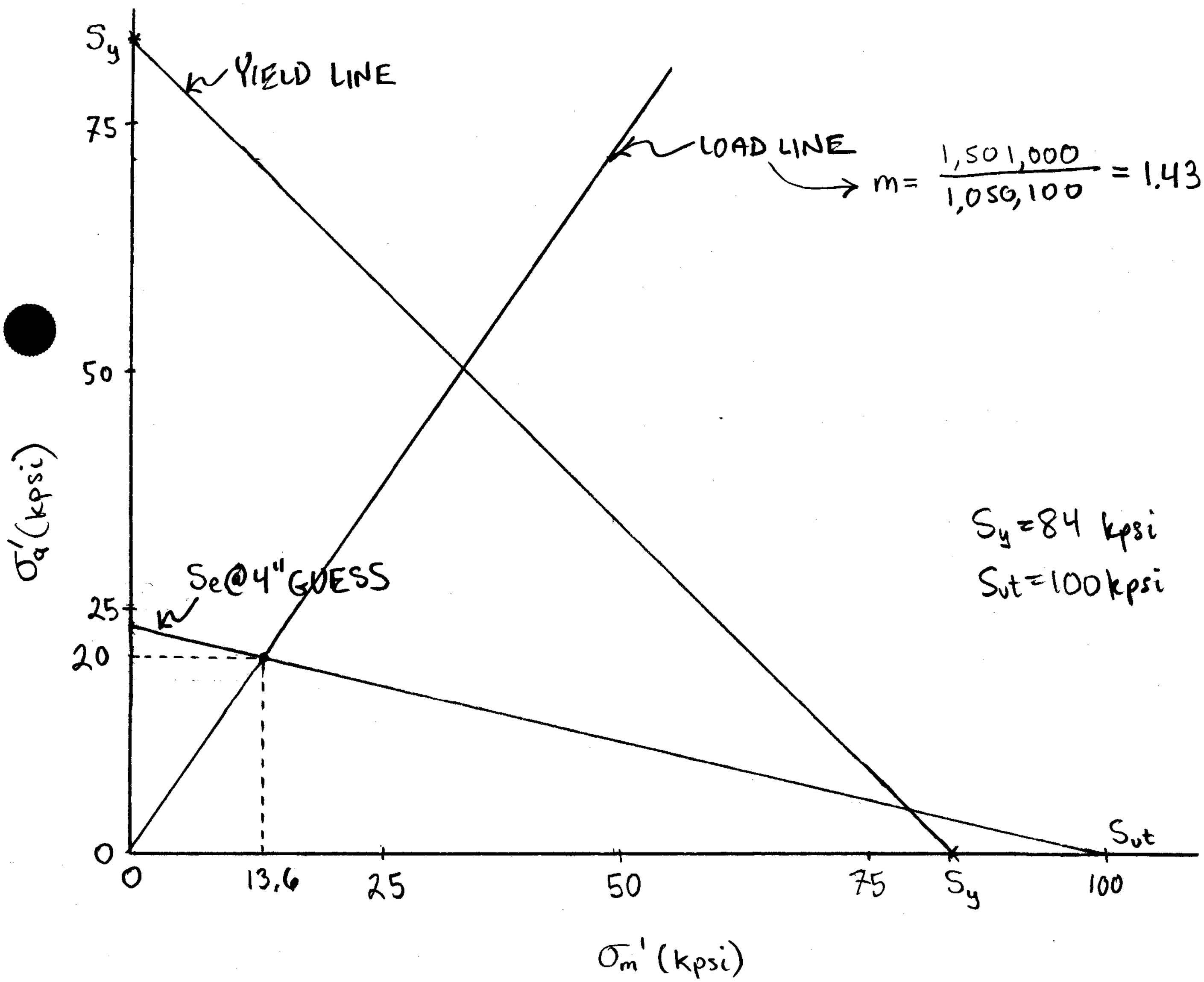
LAB # 3 - FATIGUE (CONT'D)

$C_{TEMP} = 1$ (EQN 6.7f)

$C_{RELIAB} = 0.814$ (TABLE 6-4)

$S_e = \frac{1}{2} 100\,ksi\,(1)\,0.76\,(0.76)\,1\,(0.814) = 23{,}508$ psi

AND NOW, THE MODIFIED-GOODMAN DIAGRAM



FROM GRAPH USING S_e @ 4" GUESS, $\sigma'_{a,allow} = 20{,}000$ psi

SINCE THE FACTOR OF SAFETY IN FATIGUE IS 1.5, THE MAX ALLOWABLE ALT STRESS IS ACTUALLY 20 kpsi/1.5 = 13,333 psi

SET THIS EQUAL TO THE EXPRESSION FOR ALT STRESS & SOLVE FOR DIAMETER.

$$\frac{1{,}501{,}000 \text{ in-lb}}{d^3} = \sigma'_a = 13{,}333 \text{ psi}$$

$$d = \sqrt[3]{\frac{1{,}501{,}000}{13{,}333}} = 4.83 \text{ in}$$

THE GUESS OF 4 in IS CLOSE, BUT ANOTHER ITERATION MAY BE JUSTIFIED.

FIND S_e @ 4.8" → $C_{size} = 0.869\, d^{-0.097} = 0.746$

& S_e @ 4.8" GUESS = 23,085 (TOO CLOSE TO 23,508 psi)

AN ADDITIONAL ITERATION WILL NOT YIELD ANY MEANINGFUL INFO.

$d = 4.8$ in

FINALLY, WE DOUBLE CHECK OUR ASSUMPTIONS TO NEGLECT AXIAL AND TRANSVERSE SHEAR STRESS.

AXIAL $\sigma = \frac{P}{A} = \frac{3695.3 \text{ lb}}{\frac{\pi}{4}(4.8 \text{ in})^2} = 204$ psi (NEGLIGIBLE COMPARED TO 12,000)

TRANSVERSE $\tau = \frac{4}{3}\frac{V}{A} = \frac{4(3288.6 \text{ lb})}{3\,\frac{\pi}{4}(4.8 \text{ in})^2} = 242$ psi (NEGLIGIBLE)

CALCULATE FOS IN YIELD...

$$\text{MAX EQUIVALENT STRESS} = \sqrt{(13{,}333)^2 + 3(5{,}482)^2} = 16{,}368 \text{ psi}$$

$$FOS = \frac{84{,}000 \text{ psi}}{16{,}368 \text{ psi}} = 5.1 \rightarrow MS = 1 - 5.1 = 4.1 > 0.25 \checkmark$$

OPTIONAL DIAMETER DOUBLE CHECK (USING σ'_m ALLOWABLE, FROM PLOT)

$$\frac{1{,}050{,}100}{d^3} = \sigma'_m = \frac{13{,}600}{1.5} \rightarrow d = 4.86$$

NEED BETTER GOODMAN PLOT TO BE MORE ACCURATE. (NOT NEEDED FOR THIS EXERCISE.)