9-5 NORTON

FROM 9-2

$$R_1 = P(b-a)\left[1 - \frac{1}{\ell}\left(a + \frac{b-a}{2}\right)\right] \quad \& \quad R_2 = \frac{P(b-a)\left(a + \frac{b-a}{2}\right)}{\ell}$$

AND

$$q = R_1\langle x-0\rangle^{-1} - P\langle x-a\rangle^0 + P\langle x-b\rangle^0 + R_2\langle x-\ell\rangle^{-1}$$

$$V = \int q\,dx = R_1\langle x-0\rangle^0 - P\langle x-a\rangle^1 + P\langle x-b\rangle^1 + R_2\langle x-\ell\rangle^0$$

$$M = \int V\,dx = R_1\langle x-0\rangle^1 - \frac{P}{2}\langle x-a\rangle^2 + \frac{P}{2}\langle x-b\rangle^2 + R_2\langle x-\ell\rangle^1$$

NOW USING THE ABOVE, WE FIND THE FORMULA FOR VERTICAL DEFLECTION.

$$\text{SLOPE } \theta = \int \frac{M}{EI}\,dx = \frac{1}{EI}\left\{\frac{R_1}{2}\langle x-0\rangle^2 - \frac{P}{6}\langle x-a\rangle^3 + \frac{P}{6}\langle x-b\rangle^3 + \frac{R_2}{2}\langle x-\ell\rangle^2 + C_3\right\}$$

FINALLY, THE DEFLECTION $\underline{y}$

$$y = \int \theta\,dx = \frac{1}{EI}\left\{\frac{R_1}{6}\langle x-0\rangle^3 - \frac{P}{24}\langle x-a\rangle^4 + \frac{P}{24}\langle x-b\rangle^4 + \frac{R_2}{6}\langle x-\ell\rangle^3 + C_3x + C_4\right\}$$

USING $y = 0$ WHEN $x = 0$

$$0 = \frac{1}{EI}\left\{\frac{R_1}{6}(0) - \frac{P}{24}(0) + \frac{P}{24}(0) + \frac{R_2}{6}(0) + C_3(0) + C_4\right\}$$

THUS $C_4 = 0$

USING $y = 0$ WHEN $x = \ell$

$$0 = \frac{1}{EI}\left\{\frac{R_1}{6}\ell^3 - \frac{P}{24}(\ell-a)^4 + \frac{P}{24}(\ell-b)^4 + \frac{R_2}{6}(0) + C_3\ell\right\}$$

$$C_3 = \frac{1}{\ell}\left\{-\frac{R_1}{6}\ell^3 + \frac{P}{24}(\ell-a)^4 - \frac{P}{24}(\ell-b)^4\right\}$$

9-5 NORTON (CONT'D)

MAX DEFLECTION OCCURS WHERE $\theta = 0$

$$0 = \left\{ \frac{R_1}{2}\langle x-0\rangle^2 - \frac{P}{6}\langle x-a\rangle^3 + \frac{P}{6}\langle x-b\rangle^3 + \frac{R_2}{2}\langle x-l\rangle^2 + C_3 \right\}$$

TORSIONAL DEFLECTION IS FOUND FROM

$$\phi = \frac{Tl}{GJ}$$

HAND CALCULATIONS FOR ROW a:

$R_1 = 300\,N$
$R_2 = 1700\,N$ } SEE PROBLEM 9-2 SOLUTION

$$X_{\substack{MAX\\DEF}} = \sqrt{\frac{-2C_3}{R_1}} = 0.114\,m$$

$$C_3 = \frac{1}{.2m}\left\{ -\frac{300N}{6}(.2m)^3 + \frac{100{,}000\,N/m}{24}(.2m-.16m)^4 - \frac{100{,}000\,N/m}{24}(.2m-.18m)^4 \right.$$

$$= -1.95\,N/m^2$$

$$y_{max} = \frac{64}{207\times10^9 N/m^2 (\pi)(.04m)^4}\left\{ \frac{300N}{6}(.114m)^3 - 1.95\,N/m^2(.114m) \right\}$$

$$y_{max} = -0.000005698\,m = -0.0057\,mm$$

$$\phi = \frac{Tl}{GJ} = \frac{2000\,Nm(.2m)\,32}{80.8\times10^9 N/m^2(\pi)\,.04^4 m} = 0.0197\,rad = 1.129°$$

9.5

Row	l (cm)	a (cm)	b (cm)	P (N/m)	Tmin (Nm)	Tmax (Nm)	R1 (N)	R2 (N)	C3 (N/m^2)	X (m)	Ymax (mm)	Theta (degrees)
a	0.2	0.16	0.18	100000	0	2000	300	1700	-1.95	0.11402	-0.0057	1.129
b	0.12	0.02	0.07	50000	-100	600	1563	938	-2.12	0.05711	-0.0029	0.203
c	0.14	0.04	0.12	75000	-200	400	2571	3429	-6.17	0.07143	-0.0111	0.158
d	0.08	0.04	0.08	100000	0	2000	1000	3000	-0.93	0.04322	-0.0010	0.451
e	0.17	0.06	0.12	150000	-200	500	4235	4765	-15.25	0.08608	-0.0333	0.240
f	0.24	0.16	0.22	75000	1000	2000	938	3563	-8.47	0.13441	-0.0292	1.354

NOTE: The values shown above for torsional deflection differ from those of the solution manual because the text uses G = 72 GPa, while the analysis above uses G = 80.8 GPa since the shaft is steel as per the problem statement.