MIN LOAD = 175 lb
MAX LOAD = 225 lb
WORKING DEFLECTION = 0.85 in
ENDS SQUARED AND GROUNDED
UNPEELED MUSIC WIRE - A228
10% CLASH ALLOWANCE
FORCING FREQUENCY IS 500 rpm
DESIGN FOR INFINITE LIFE
MINIMIZE PACKAGE SIZE
FIND F.O.S. AGAINST FATIGUE, YIELD, AND SURGE

\[ k = \frac{225 - 175}{0.85} = 58.82 \text{ lb/in} \]

\[ k = \frac{dG}{8C^3N_a} \]

CHOICE OF C & d, D & d, OR C & D WILL
DETERMINE PHYSICAL SIZE OF SPRING. "MINIMIZE
PACKAGE SIZE" IS A BIT VAGUE. THE "PACKAGE SIZE"
CAN BE CONSIDERED TO BE THE INSTALLED SIZE
OF THE SPRING WHICH WOULD BE \[ [(1+10\%)(0.85\text{in})+L_s] \times \]
d + d. \[ C = \frac{D}{d}, \quad L_s = d(N_a+2). \]
C MUST BE BETWEEN
4 & 12, AND d MUST BE BETWEEN 0.004 in & 0.312 in
MINIMUM INSTALLED VOLUME IS ABOUT 28 in^3 @ d = 0.25 in
& C = 9. (FROM SPREADSHEET) [ASSUMING REASONABLE F.O.S IN
YIELD]

CALCULATE F.O.S. AGAINST YIELD,

\[ C_{\text{max}} = k_s \frac{8FC}{\pi d^2} = \left(1 + \frac{0.5}{9}\right) \frac{8(225\text{ lb})9}{\pi (0.25\text{ in})^2} = 87 \text{ kpsi} \]

\[ S_{\text{ult}} = 225 \text{ kpsi} \quad S_{\text{sys}} = 0.45(225 \text{ kpsi}) = 101.25 \text{ kpsi} \]

\[ \text{F.O.S. yield} = \frac{101}{87} = 1.16 \]
CALCULATE F.O.S. IN FATIGUE

\[ F_t = 175, \quad F_{\text{max}} = 225, \quad F_m = \frac{225 + 125}{2} = 200, \quad F_a = \frac{225 - 125}{2} = 25 \]

\[ \tau_i = k_s \frac{8F_i C}{\pi d^2} = \left(1 + \frac{0.5}{9}\right) \frac{8(175)9}{\pi (0.25\text{in})^2} = 67.7 \text{ kpsi} \]

\[ \tau_m = k_s \frac{8F_m C}{\pi d^2} = \left(1 + \frac{0.5}{9}\right) \frac{8(200)9}{\pi (0.25\text{in})^2} = 77.4 \text{ kpsi} \]

\[ \tau_a = k_w \frac{8F_a C}{\pi d^2} = 1.16 \frac{8(25)9}{\pi (0.25\text{in})^2} = 10.7 \text{ kpsi} \]

\[ k_w = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{35}{32} + \frac{0.615}{c} = 1.16 \]

\[ \text{Ses} = 0.707 \frac{\text{Sew} \cdot \text{Sus}}{\text{Sus} - 0.707 \text{Sew}} = 0.707 \frac{45(15)}{151 - 0.707(45)} = 40.3 \text{ kpsi} \]

\[ \text{Sew} = 45 \text{ kpsi} \quad \text{(unpeened, eqn 13.12)} \]

\[ \text{Sus} = 0.67(\text{Sus}) = 0.67(225 \text{ kpsi}) = 151 \text{ kpsi} \quad \text{(eqn 13.4)} \]

\[ \text{FOS}_{\text{fatigue}} = \frac{\text{Ses} \cdot \tau_i}{\text{Ses} \cdot \tau_m - \tau_i} + \text{Sus} \cdot \tau_a = \frac{40.3(151 - 67.7)}{40.3(77.4 - 67.7) + 151(10.7)} = 1.67 \]

CALCULATE F.O.S. AGAINST SURGE

\[ f = \frac{1}{2} \sqrt{\frac{\frac{8}{W_a}}{\pi^2 \lambda D^2} \cdot \frac{(386 \text{ in/sec})}{0.8411 \text{ lb}}} = 82 \text{ Hz} \]

\[ W_a = \frac{\pi^2 D^2 \lambda}{4} = \frac{\pi^2 (2.25 \text{ in})^2 (2.25 \text{ in}) 8.5 (0.285 \text{ lb/in}^3)}{4} = 0.8411 \text{ lb} \]

\[ \text{F.O.S. surge} = \frac{82}{13(8.3)} = \frac{82}{108} = 0.75 \]

Technically, the spring should be redesigned with less active coils!
Ideally, the spring should be redesigned with less oscillating mass, but since the natural frequency is 10x the forcing frequency (13 is desired), we’ll let it go.

Spring dimensions are

\[ d = 0.25 \text{ in} \]

\[ D = Cd = 9(0.25) = 2.25 \text{ in} \]

\[ N_a = \frac{dG}{8C^3k} = \frac{0.25 \text{ in}(11,700 \times 10^6 \text{ psi})}{8(9)^3 \times 58.82 \text{ lb/in}} = 8.5 \text{ (nearest 1/4)} \]

\[ L_s = d(N_a + 2) = 0.25(8.5 + 2) = 2.625 \text{ in} \]

\[ y_c = 0.085 \text{ in} \]

\[ y_w = 0.85 \text{ in} \]

\[ y_c = \frac{175 \times 16}{58.82 \text{ lb/in}} = 2.975 \text{ in} \]

\[ L_f = L_s + y_c + y_w + y_c = 6.14 \text{ in} \]

installed length = \[ L_s + y_c + y_w = 3.56 \text{ in} \]