NORTON 13.19

MIN LOAD = 175 lb
MAX LOAD = 225 lb
WORKING DEFLECTION = 0.85 in
ENDS SQUARED AND GROUND
UNPEENED MUSIC WIRE – A228
10% CLASH ALLOWANCE
FORCING FREQUENCY IS 500 rpm
DESIGN FOR INFINITE LIFE
MINIMIZE PACKAGE SIZE
FIND F.O.S. AGAINST FATIGUE, YIELD, AND SURGE

$$k = \frac{225 - 175}{0.85} = 58.82 \text{ lb/in}$$

$$k = \frac{dG}{8C^3 N_a}$$

CHOICE OF C & d, D & d, OR C & D WILL DETERMINE PHYSICAL SIZE OF SPRING. "MINIMIZE PACKAGE SIZE" IS A BIT VAGUE. THE "PACKAGE SIZE" CAN BE CONSIDERED TO BE THE INSTALLED SIZE OF THE SPRING WHICH WOULD BE $[(1+10\%)(0.85\text{in}) + L_s] \times [D+d]$. $C = \frac{D}{d}$, $L_s = d(N_a + 2)$. C MUST BE BETWEEN 4 & 12, AND d MUST BE BETWEEN 0.004 in & 0.312 in

MINIMUM INSTALLED VOLUME IS ABOUT 28 in^3 @ $d = 0.25$ in & $C = 9$. (FROM SPREADSHEET) [ASSUMING REASONABLE FOS IN YIELD]

CALCULATE F.O.S. AGAINST YIELD.

$$\tau_{max} = K_s \frac{8FC}{\pi d^2} = \left(1 + \frac{0.5}{9}\right) \frac{8(225\,\text{lb})\,9}{\pi (0.25\,\text{in})^2} = 87 \text{ kpsi}$$

$S_{ut} = 225$ kpsi, $S_{ys} = 0.45(225 \text{ kpsi}) = 101{,}250$ kpsi

$$\text{F.O.S.}_{yield} = \frac{101}{87} = 1.16$$

NORTON 13,19 CONT'D

CALCULATE F.O.S. IN FATIGUE

$F_i = 175$, $F_{max} = 225$, $F_m = \frac{225+175}{2} = 200$, $F_a = \frac{225-175}{2} = 25$

$$\tau_i = K_s \frac{8 F_i C}{\pi d^2} = \left(1 + \frac{0.5}{9}\right) \frac{8(175)9}{\pi (0.25\,in)^2} = 67.7\ kpsi$$

$$\tau_m = K_s \frac{8 F_m C}{\pi d^2} = \left(1 + \frac{0.5}{9}\right) \frac{8(200)9}{\pi (0.25\,in)^2} = 77.4\ kpsi$$

$$\tau_a = K_w \frac{8 F_m C}{\pi d^2} = 1.16 \frac{8(25)9}{\pi (0.25\,in)^2} = 10.7\ kpsi$$

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{35}{32} + \frac{0.615}{9} = 1.16$$

$$S_{es} = 0.707 \frac{S_{ew} S_{us}}{S_{us} - 0.707 S_{ew}} = 0.707 \frac{45(151)}{151 - 0.707(45)} = 40.3\ kpsi$$

$S_{ew} = 45$ kpsi (UNPEENED, EQN 13.12)

$S_{us} = 0.67(S_{us}) = 0.67(225\ kpsi) = 151$ kpsi (EQN 13.4)

$$FOS_{FATIGUE} = \frac{S_{es}(S_{us} - \tau_i)}{S_{es}(\tau_m - \tau_i) + S_{us}\tau_a} = \frac{40.3(151 - 67.7)}{40.3(77.4 - 67.7) + 151((10.7)} = 1.67$$

CALCULATE F.O.S. AGAINST SURGE

$$f = \frac{1}{2}\sqrt{\frac{kg}{w_a}} = \frac{1}{2}\sqrt{\frac{58.82\,lb/in\,(386\,in/sec^2)}{0.841\,lb}} = 82\ Hz$$

$$w_a = \frac{\pi^2 d^2 D N_a \gamma}{4} = \frac{\pi^2 (.25\,in)^2 (2.25\,in)\, 8.5\, (.285\,lb/in^3)}{4} = 0.841\,lb$$

$$F.O.S._{SURGE} = \frac{82}{13(8.3)} = \frac{82}{108} = 0.75$$

TECHNICALLY, THE SPRING SHOULD BE REDESIGNED WITH LESS ACTIVE COILS!

NORTON 13.19 CONT'D

IDEALLY, THE SPRING SHOULD BE REDESIGNED WITH LESS OSCILLATING MASS, BUT SINCE THE NATURAL FREQUENCY IS 10X THE FORCING FREQUENCY (13 IS DESIRED), WE'LL LET IT GO.

SPRING DIMENSIONS ARE

$d = 0.25\ in$

$D = Cd = 9(.25) = 2.25\ in$

$N_a = \frac{dG}{8C^3k} = \frac{0.25in(11.7\times10^6\ psi)}{8(9)^3\ 58.82\ lb/in} = 8.5$ (NEAREST 1/4)

$L_s = d(N_a+2) = 0.25(8.5+2) = 2.625\ in$

$y_c = 0.085\ in$

$y_w = 0.85\ in$

$y_i = \frac{175\ lb}{58.82\ lb/in} = 2.975\ in$

$L_f = L_s + y_c + y_w + y_i = 6.54\ in$

installed length $= L_s + y_c + y_w = 3.56\ in$