A countershaft with two spur gears, is shown below. The gears are made of 4140 steel, quenched and tempered at 1200F and the shaft of a steel with a BHN of 450. The yield strength of the material is equal to 80% of the ultimate strength. The factor of safety in fatigue is to be 2.75. Static (theoretical) stress concentration factors are 2.50 in bending, 1.75 in axial, and 2.25 in torsional loadings. Notch radii are 0.25 inches.

(1) Verify the FBD shown is correct and correct it, if necessary. Show all calculations necessary to justify your decisions.
(2) Construct the shear, moment, axial load, and torsional load diagrams for the shaft.
(3) Using the modified-Goodman line, determine the diameter of the shaft, to the nearest cm, at the critical section of the shaft.

If needed, use the following equations to determine the transmitted (tangential) and radial loads.

\[
F_{radial} = F_{transmitted} \tan \phi \\
F_{transmitted} = F \cos \phi \\
\phi = 20^\circ
\]

(1) 15 points
(2) 15 points
(3) 20 points

Possible points: 50
\[ \sum F_x = 0 \rightarrow O = 0 \]
\[ \sum F_y = 0 \rightarrow D_y - 4.104 \text{ kN} - 8.208 \text{ kN} + C_y = 0 \]
\[ \sum F_z = 0 \rightarrow D_z + 11.276 \text{ kN} - 22.553 \text{ kN} + C_z = 0 \]
\[ \sum M_y = 0 \rightarrow 3947 \text{ in-lb} = 3947 \text{ in-lb} \]
\[ \sum M_y = 0 \rightarrow -11.276 \text{ kN} \cdot (450 \text{ mm}) + 22.553 \text{ kN} \cdot (700 \text{ mm}) - C_z(1.15 \text{ m}) = 0 \]
\[ C_z = \frac{-11.276 \text{ kN} \cdot (450 \text{ mm}) + 22.553 \text{ kN} \cdot (700 \text{ mm})}{1.15 \text{ m}} = 9.315 \text{ kN} \]
\[ \sum M_z = 0 \rightarrow -4.104 \text{ kN} \cdot (450 \text{ mm}) - 8.208 \text{ kN} \cdot (700 \text{ mm}) + C_y(1.15 \text{ m}) = 0 \]
\[ C_y = \frac{-4.104 \text{ kN} \cdot (450 \text{ mm}) + 8.208 \text{ kN} \cdot (700 \text{ mm})}{1.15 \text{ m}} = 6.602 \text{ kN} \]

\[ D_y = 4.140 \text{ kN} + 8.208 \text{ kN} - 6.602 \text{ kN} = 5.746 \text{ kN} \]
\[ D_z = -11.276 \text{ kN} + 22.553 \text{ kN} - 9.315 \text{ kN} = 1.962 \text{ kN} \]
SHEAR & MOMENT DIAGRAMS:

VERTICAL PLANE (x-y)

4.104 kN

5.746 kN

5.746 kN

1642 kN

2.586 kN-m

6.602 kN

M (kN-m)

8.208 kN

B

2.996 kN-m

C

HORIZONTAL PLANE (x-z)

22.553 kN

9.315 kN

1.962 kN

1.962 kN

13.228 kN

0.883 kN-m

4.192 kN-m

TORQUE DIAGRAM

3.947 kN-m

NO AXIAL LOADS!
CRITICAL POINT: B (LEFT HAND SIDE)

\[ M = \sqrt{(2.996 \text{ KNm})^2 + (4.192 \text{ KNm})^2} = 5.15 \text{ KNm} \]
\[ T = 3.947 \text{ KNm} \]

STRESSES:

ALTERNATING (BENDING)

\[ \sigma_x = \frac{32M}{\pi d^3} K_f = \frac{32 (5.150 \text{ Nm})}{\pi d^3} (2.44) = \frac{127.996 \text{ Nm}}{d^3} \]
\[ K_f = 1 + \phi (K_t - 1) = 1 + 0.96 (2.5 - 1) = 2.44 \]
\[ \sigma^i_t = \frac{127.996 \text{ Nm}}{d^3} \quad \text{(VON MISES)} \]

MEAN STRESS (TORSION)

\[ \tau_x = \frac{16T}{\pi d^3} K_t = \frac{16 (3.947 \text{ Nm})}{\pi d^3} (2.125) = \frac{45.229 \text{ Nm}}{d^3} \]
\[ \sigma_m = \sqrt{\frac{1}{3} \left( \frac{45.229 \text{ Nm}}{d^3} \right)^2} = \frac{78.339 \text{ Nm}}{d^3} \quad \text{(VON MISES)} \]

ENDURANCE LIMIT:

\[ S_u = 500 \text{ (BHN)} = 500 \text{ (450)} = 225 \text{ kpsi} = 1551 \text{ MPa} \]
\[ S_e' = 700 \text{ MPa} \]

\[ S_e = C_{load} C_{size} C_{surf} C_{temp} C_{reliab} S_e' \]
\[ C_{load} = 1 \]
\[ C_{size} = 1.184 d^{-0.097} = 1.184 (50 \text{ mm})^{-0.097} = 0.81 \quad \text{(GUESS 50 mm)} \]
\[ C_{surf} = 0.59 \quad \text{(MACHINED)} \]
\[ C_{temp} = 1 \]
\[ C_{reliab} = 1 \]
\[ S_e = 1 (0.81) (0.59) (1) (700 \text{ MPa}) = 336 \text{ MPa} \]
THE MODIFIED GOODMAN DIAGRAM

\[ m = \frac{S_y}{S_t} = 1.63 \]

\[ S_t = 1,551 \text{ MPa} \]
\[ S_y = 1,241 \text{ MPa} \]
\[ S_e = 336 \text{ MPa} \quad (50 \text{ mm GUESS}) \]

\[ \sigma_a'_{\text{allow}} = \frac{300 \text{ MPa}}{2.75} \rightarrow d = \sqrt[3]{\frac{127,996 \text{ Nm}}{109,000,000 \text{ Pa}}} = 0.106 \text{ m} = 106\text{mm} \]

RE-DRAW GOODMAN LINE @ 110 mm SIZE CORRECTION → \( S_e = 313 \text{ MPa} \)

Now, \( \sigma_y'_{\text{allow}} = \frac{275 \text{ MPa}}{2.75} \rightarrow d = \sqrt[3]{\frac{127,996 \text{ Nm}}{100,000,000 \text{ Pa}}} = 0.109 \text{ m} = 109\text{mm} \)

\[ d = 11 \text{ cm} \]

DOUBLE-CHECK: \( \sigma_y'_{\text{allow}} = \frac{170 \text{ MPa}}{2.75} \rightarrow d = \sqrt[3]{\frac{78,339 \text{ Nm}}{62,000,000 \text{ Pa}}} = 108\text{mm} \)