MACHINE DESIGN WINTER TERM - 1999 EXAM -01

NAME:_____________________________ January 28, 1999

- Read the problem carefully.
- Work must be neat and orderly to be graded.
- Return this page with your exam, perform no work on this page. Any work performed on this page will not be graded.

A countershaft with two spur gears, is shown below. The gears are made of 4140 steel, quenched and tempered at 1200F and the shaft of a steel with a BHN of 450. The yield strength of the material is equal to 80% of the ultimate strength. The factor of safety in fatigue is to be 2.75. Static (theoretical) stress concentration factors are 2.50 in bending, 1.75 in axial, and 2.25 in torsional loadings. Notch radii are 0.25 inches.

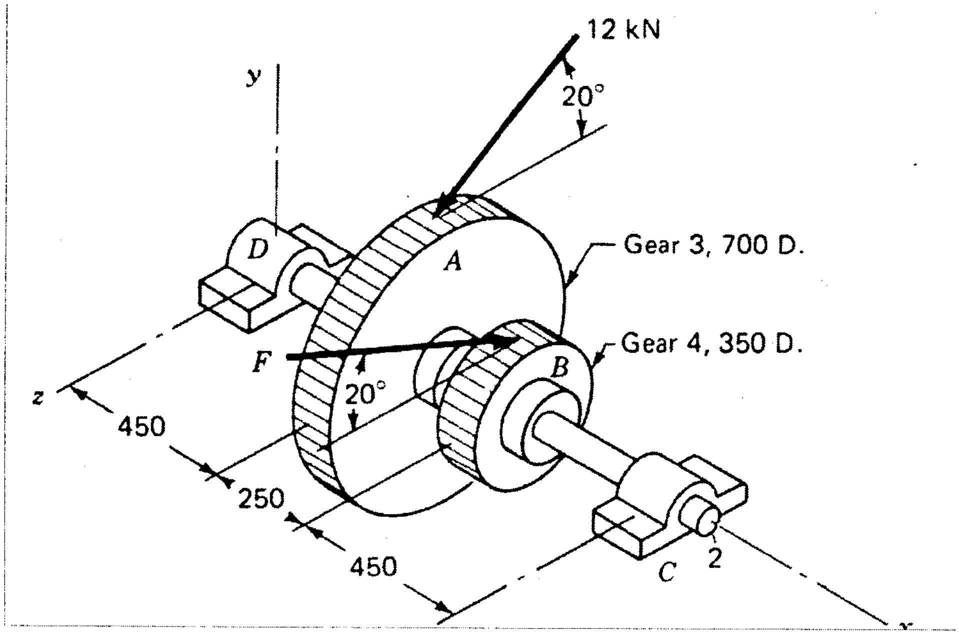
(1) Verify the FBD shown is correct and correct it, if necessary. Show all calculations necessary to justify your decisions.
(2) Construct the shear, moment, axial load, and torsional load diagrams for the shaft.
(3) Using the modified-Goodman line, determine the diameter of the shaft, to the nearest cm, at the critical section of the shaft.

If needed, use the following equations to determine the transmitted (tangential) and radial loads.

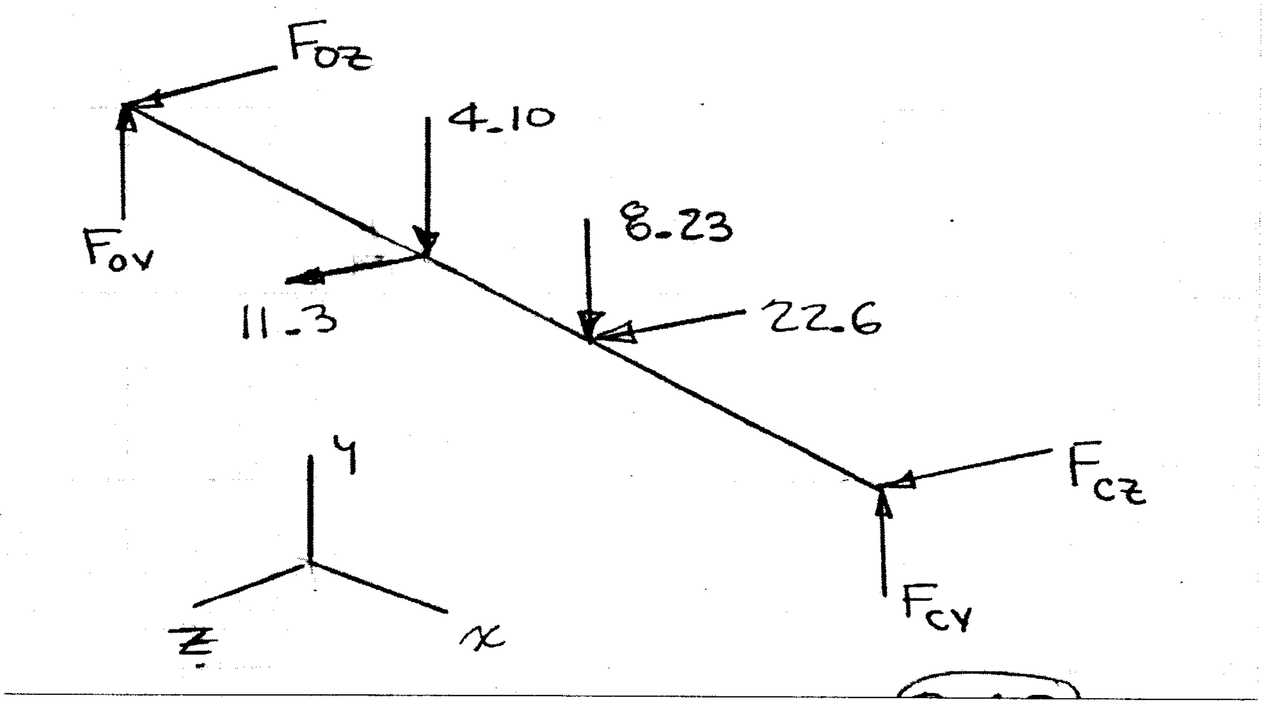
$$F_{radial} = F_{transmitted} \tan\varphi$$
$$F_{transmitted} = F\cos\phi$$
$$\phi = 20^0$$

(1) 15 points
(2) 15 points
(3) 20 points

Possible points: 50



12 kN
20°
y
D
A
Gear 3, 700 D.
F
Gear 4, 350 D.
B
20°
z
450
250
450
C
2
x



F_{OZ}
4.10
F_{OY}
8.23
11.3
22.6
Y
F_{CZ}
Z
x
F_{CY}

EXAM #1 1 pm

ME 340
1/28/99
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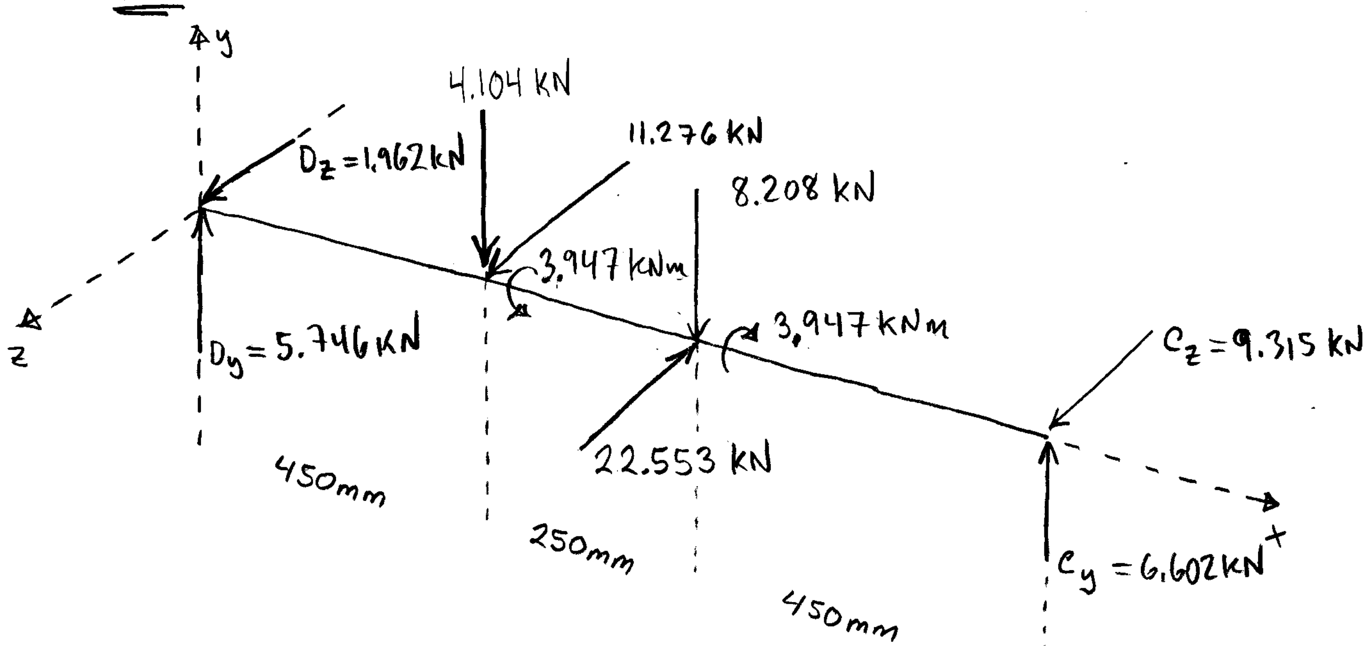
FBD1



STATICS:

$\Sigma F_x = 0 \rightarrow 0 = 0$

$\Sigma F_y = 0 \rightarrow D_y - 4.104\text{kN} - 8.208\text{kN} + C_y = 0$

$\Sigma F_z = 0 \rightarrow D_z + 11.276\text{kN} - 22.553\text{kN} + C_z = 0$

$\Sigma M_x^D = 0 \rightarrow 3947\text{in-lb} = 3947\text{in-lb}$

$\Sigma M_y^D = 0 \rightarrow -11.276\text{kN}(.450\text{m}) + 22.553\text{kN}(.700\text{m}) - C_z(1.15\text{m}) = 0$

$$C_z = \frac{-11.276\text{kN}(.450\text{m}) + 22.553\text{kN}(.700\text{m})}{1.15\text{m}} = 9.315\text{kN}$$

$\Sigma M_z^D = 0 \rightarrow -4.104\text{kN}(.450\text{m}) - 8.208\text{kN}(.700\text{m}) + C_y(1.15\text{m}) = 0$

$$C_y = \frac{+4.104(.450\text{m}) + 8.208\text{kN}(.700\text{m})}{1.15\text{m}} = 6.602\text{kN}$$

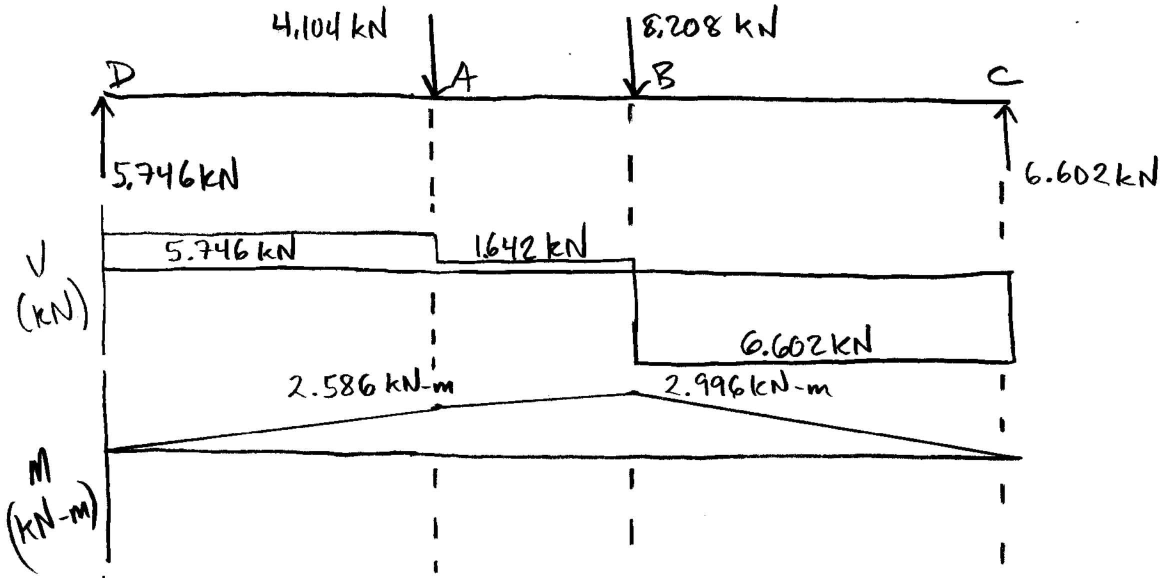
$D_y = 4.140\text{kN} + 8.208\text{kN} - 6.602\text{KN} = 5.746\text{kN}$

$D_z = -11.276\text{kN} + 22.553\text{kN} - 9.315\text{kN} = 1.962\text{kN}$

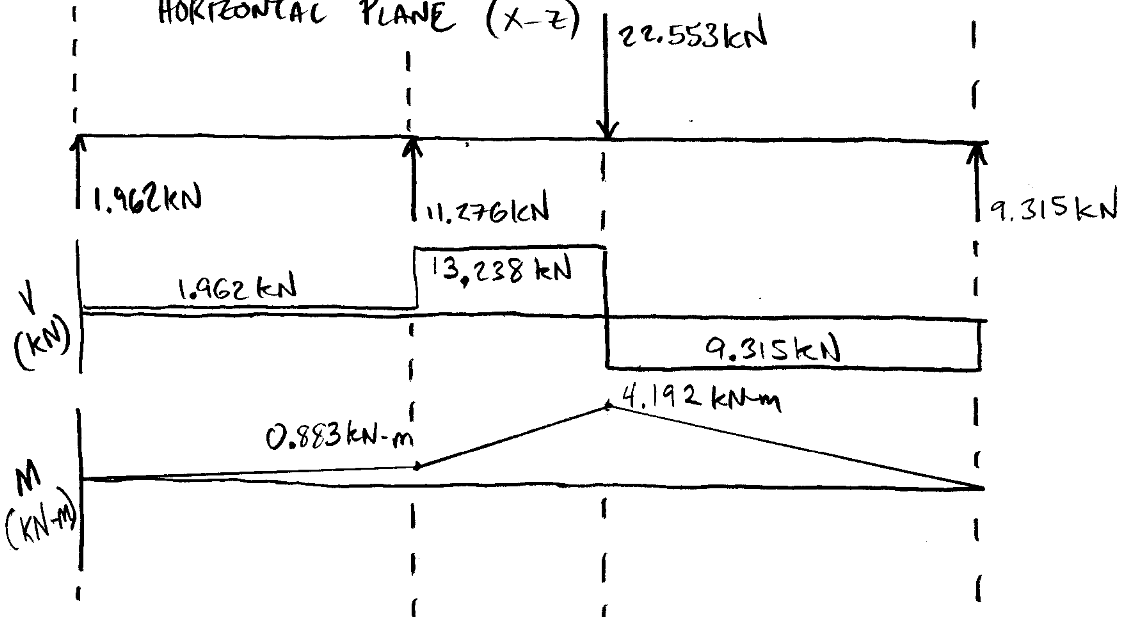
EXAM #1 CONT'D 1PM

SHEAR & MOMENT DIAGRAMS:

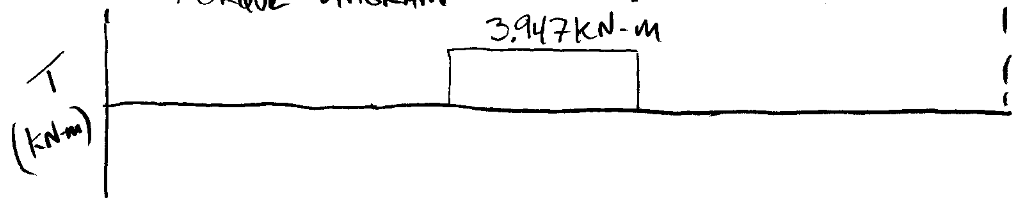
VERTICAL PLANE (x-y)



HORIZONTAL PLANE (x-z)



TORQUE DIAGRAM



NO AXIAL LOADS!

EXAM #1 CONT'D 1PM

CRITICAL POINT: B (LEFT HAND SIDE)

$$M = \sqrt{(2.996\,kNm)^2 + (4.192\,kNm)^2} = 5.15\,kNm$$

$$T = 3.947\,kNm$$

STRESSES:

ALTERNATING (BENDING)

$$\sigma_x = \frac{32M}{\pi d^3} k_f = \frac{32(5{,}150\,Nm)}{\pi\, d^3}(2.44) = \frac{127{,}996\,Nm}{d^3}$$

$$k_f = 1 + q(k_t - 1) = 1 + 0.96(2.5-1) = 2.44$$

$$\sigma_a' = \frac{127{,}996\,Nm}{d^3}$$ (VON MISES)

MEAN STRESS (TORSION)

$$\tau_{xy} = \frac{16T}{\pi d^3} k_t = \frac{16(3{,}947\,Nm)}{\pi\, d^3}(2.25) = \frac{45{,}229\,Nm}{d^3}$$

$$\sigma_m' = \frac{\sqrt{3}\; 45{,}229\,Nm}{d^3} = \frac{78{,}339\,Nm}{d^3}$$ (VON MISES)

ENDURANCE LIMIT:

$S_{ut} = 500(BHN) = 500(450) = 225\,kpsi = 1{,}551\,MPa$

$Se' = 700\,MPa$

$Se = C_{LOAD}\, C_{SIZE}\, C_{SURF}\, C_{TEMP}\, C_{RELIAB}\, Se'$

$C_{LOAD} = 1$

$C_{SIZE} = 1.189\, d^{-0.097} = 1.189(50mm)^{-0.097} = 0.81$ (GUESS 50 mm)

$C_{SURF} = 0.59$ (MACHINED)

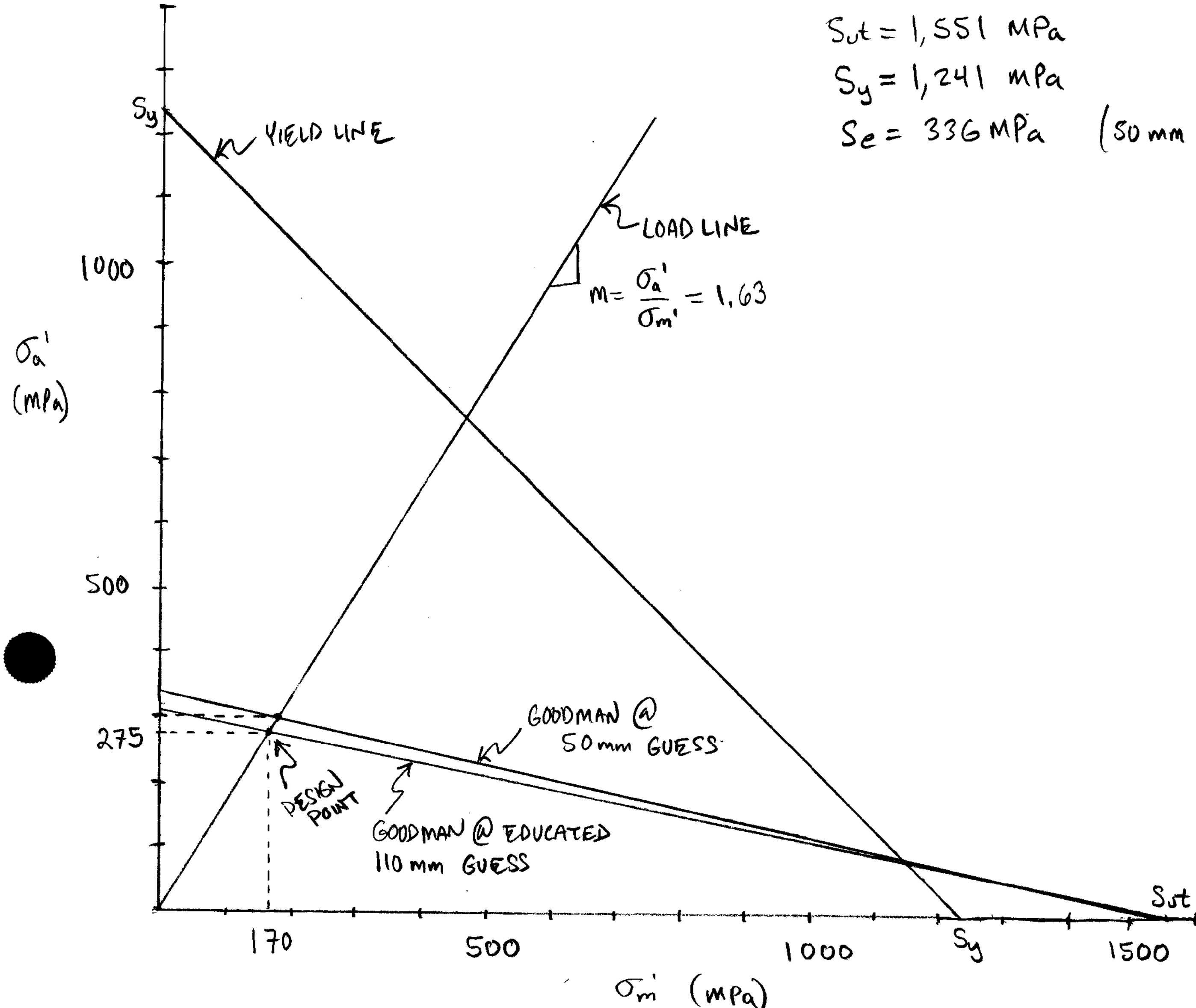
$C_{TEMP} = 1$

$C_{RELIAB} = 1$

$Se = 1(.81).59(1)1(700\,MPa) = 336\,MPa$

THE MODIFIED-GOODMAN DIAGRAM

$S_{ut} = 1{,}551$ MPa
$S_y = 1{,}241$ mPa
$S_e = 336$ MPa (50 mm GUESS)



$\sigma_{a,allow}' = \frac{300\text{ MPa}}{2.75} \rightarrow d = \sqrt[3]{\frac{127{,}996\text{ Nm}}{109{,}000{,}000\text{ Pa}}} = 0.106\text{ m} = 106\text{mm}$

RE-DRAW GOODMAN LINE @ 110 mm SIZE CORRECTION → $S_e = 313$ MPa

NOW, $\sigma_{a,allow}' = \frac{275\text{ mPa}}{2.75} \rightarrow d = \sqrt[3]{\frac{127{,}996\text{ Nm}}{100{,}000{,}000\text{ Pa}}} = 0.109\text{ m} = 109\text{mm}$

DOUBLE-CHECK: $\sigma_{m,allow}' = \frac{170\text{ MPa}}{2.75} \rightarrow d = \sqrt[3]{\frac{78{,}339\text{ Nm}}{62{,}000{,}000\text{ Pa}}} = 108\text{ mm}$

$\Big\}$ d = 11cm