

NAME: _____

January 28, 1999

- ▶ Read the problem carefully.
- ▶ Work must be neat and orderly to be graded.
- ▶ Return this page with your exam, perform no work on this page. Any work performed on this page will not be graded.

A resultant gear force, F_A , of 600 pounds acts at an angle of 20° from the y-axis of an overhanging countershaft as shown in the figure on the next page. The solid, circular machined shaft is to be made of a steel with a BHN of 360 with a yield strength equal to 75% of the ultimate strength. The factor of safety in fatigue is to be 2.50. Fatigue stress concentration factors are 2.0 in bending, 1.75 in axial, and 2.0 in torsional loadings. Notch radii are 0.38 inches.

- (1) Verify the FBD shown is correct and correct it, if necessary. Show all calculations necessary to justify your decisions.
- (2) Construct the shear, moment, axial load, and torsional load diagrams for the shaft.
- (3) Using the modified-Goodman line, determine the diameter of the shaft, to the nearest 1/8-inch, at the critical section of the shaft.

If needed, use the following equations to determine the transmitted (tangential) and radial loads.

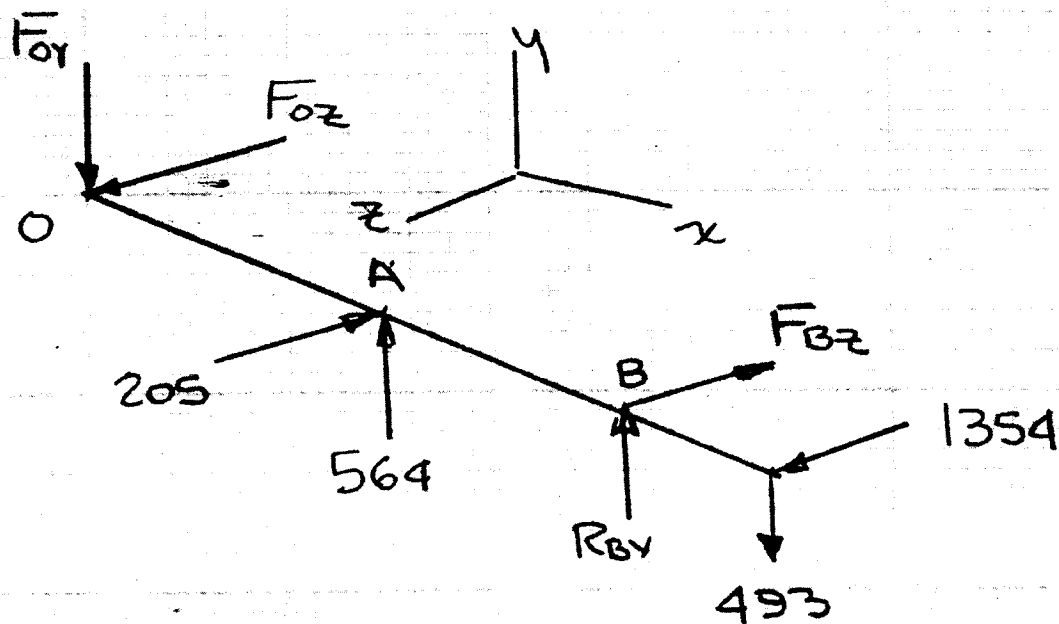
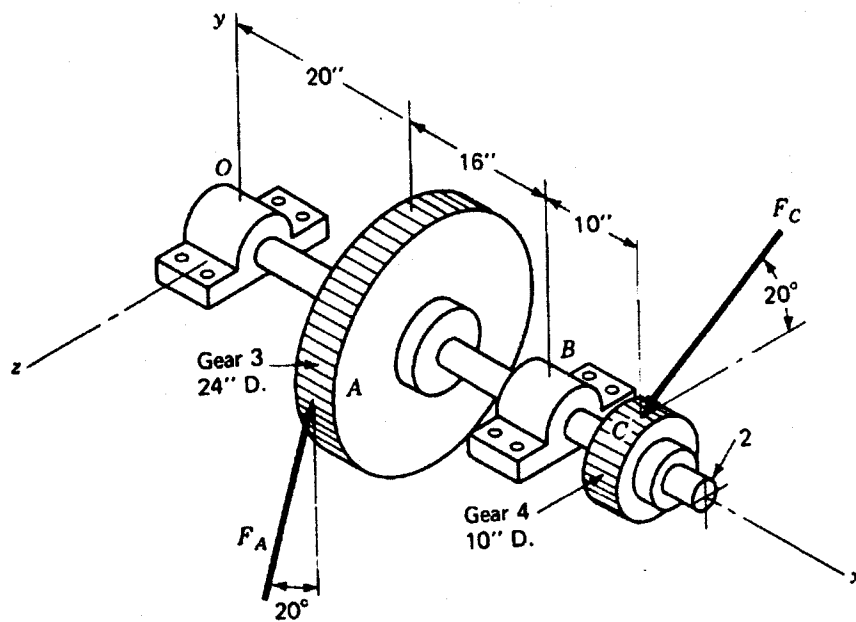
$$F_{\text{radial}} = F_{\text{transmitted}} \tan \phi$$

$$F_{\text{transmitted}} = F \cos \phi$$

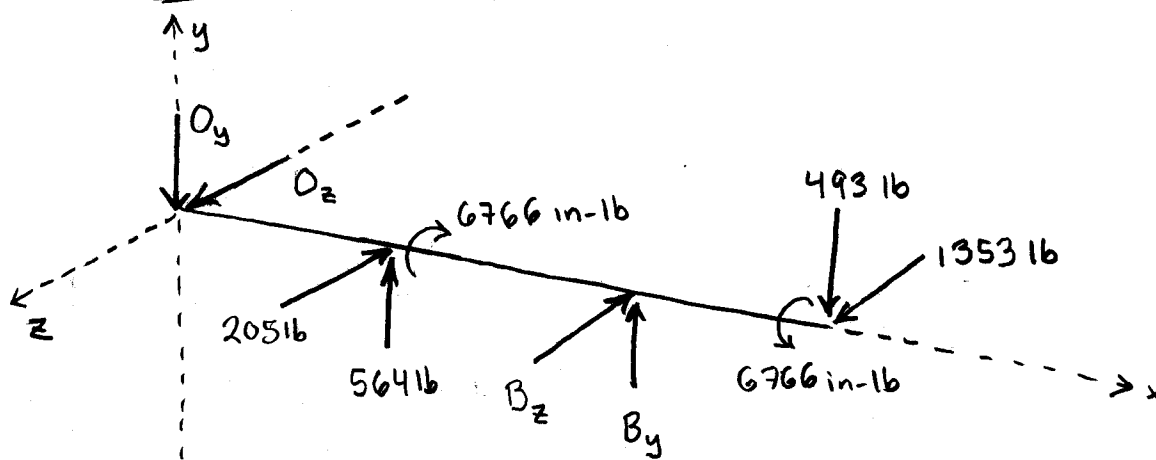
$$\phi = 20^\circ$$

- (1) 15 points
- (2) 15 points
- (3) 20 points

Possible points: 50



FBD:

STATICS:

$$\sum F_x = 0; \quad 0 = 0 \quad (\text{No AXIAL LOADS})$$

$$\sum F_y = 0; \quad -O_y + 564 \text{ lb} + B_y - 493 \text{ lb} = 0$$

$$\sum F_z = 0; \quad O_z - 205 \text{ lb} - B_z + 1354 \text{ lb} = 0$$

$$\sum M_x^0 = 0; \quad 6770 \text{ in-lb} = 6770 \text{ in-lb}$$

$$\sum M_y^0 = 0; \quad 205 \text{ lb}(20 \text{ in}) + B_z(36 \text{ in}) - 1354 \text{ lb}(46 \text{ in}) = 0$$

$$B_z = \frac{-205 \text{ lb}(20 \text{ in}) + 1353 \text{ lb}(46 \text{ in})}{36 \text{ in}} = 1615 \text{ lb}$$

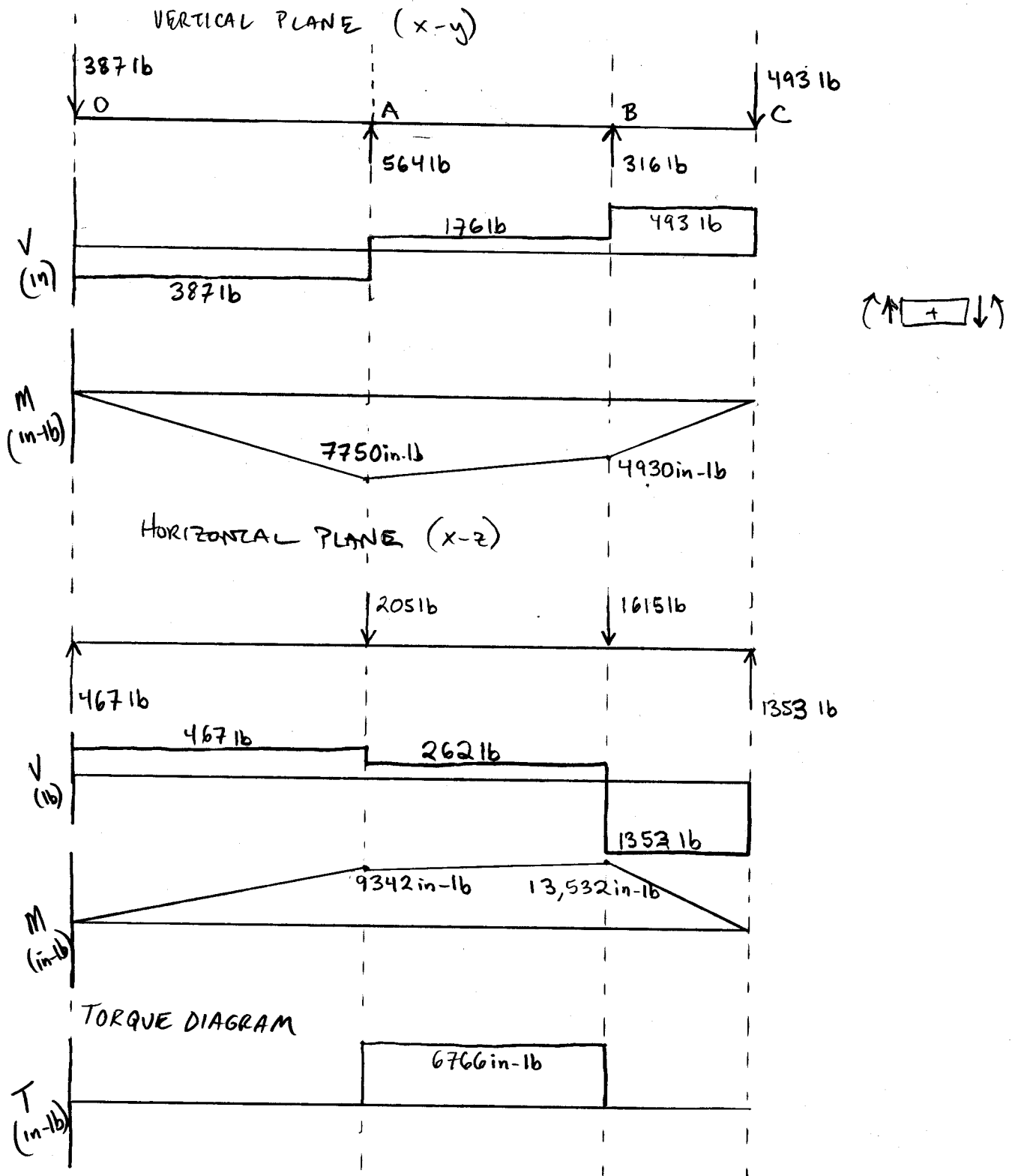
$$\sum M_z^0 = 0; \quad 564 \text{ lb}(20 \text{ in}) + B_y(36 \text{ in}) - 493 \text{ lb}(46 \text{ in}) = 0$$

$$B_y = \frac{-564 \text{ lb}(20 \text{ in}) + 493 \text{ lb}(46 \text{ in})}{36 \text{ in}} = 316 \text{ lb}$$

$$O_y = 564 \text{ lb} + 316 \text{ lb} - 493 \text{ lb} = 387 \text{ lb}$$

$$O_z = 205 \text{ lb} + 1615 \text{ lb} - 1353 \text{ lb} = 467 \text{ lb}$$

SHEAR AND MOMENT DIAGRAMS:



CRITICAL POINT IS AT B (LEFT HAND SIDE)

$$M_{\max} = \sqrt{(4930 \text{ in-lb})^2 + (13,532 \text{ in-lb})^2} = 14,402 \text{ in-lb}$$

$$T = 6766 \text{ in-lb}$$

STRESSES:

ALTERNATING (BENDING)

$$\sigma_x = \frac{32M}{\pi d^3} K_f = \frac{32(14,402 \text{ in-lb})}{\pi d^3} (2.0) = \frac{293,395}{d^3} = \sigma_a$$

$$\sigma_a' = \frac{293,395}{d^3} \quad (\text{VON MISES})$$

MEAN STRESS (TORSION)

$$\tau_{xy} = \frac{16T}{\pi d^3} K_t = \frac{16(6766 \text{ in-lb})}{\pi d^3} (2.05) = \frac{70,640}{d^3} = \tau_m$$

$$K_t = \frac{K_f - 1}{q} + 1 = \frac{2.0 - 1}{0.95} + 1 = 2.05$$

$$\sigma_m' = \frac{\sqrt{3} 70,640}{d^3} \quad (\text{VON MISES})$$

ENDURANCE LIMIT:

$$S_e = \frac{1}{2} S_{ut} C_{\text{LOAD}} C_{\text{SIZE}} C_{\text{SURF}} C_{\text{TEMP}} C_{\text{RELIAB}}$$

$$C_{\text{LOAD}} = 1$$

$$C_{\text{SIZE}} = 0.869 d^{-0.097} = 0.869 \quad (\text{GUESS 1 in})$$

$$C_{\text{SURF}} = 0.66$$

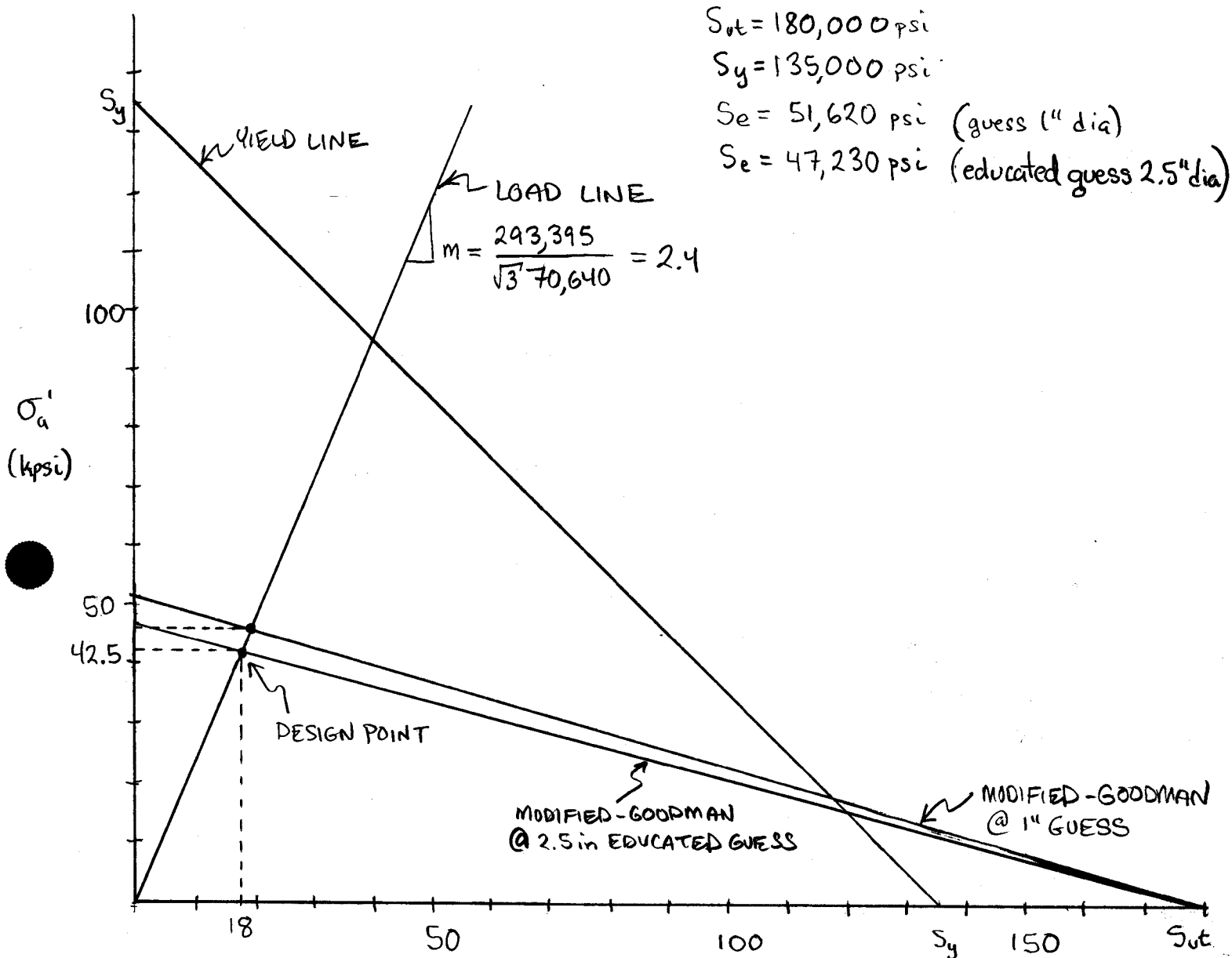
$$C_{\text{TEMP}} = 1 \quad \text{ASSUME ROOM TEMPERATURE}$$

$$C_{\text{RELIAB}} = 1 \quad \text{ASSUME 50\% RELIABILITY}$$

$$S_e = \frac{1}{2} (180,000 \text{ psi}) (1) (0.869) (0.66) (1) (1) = 51,620 \text{ psi}$$

$$[S_{ut} = 500 (\text{BHN}) = 500 (360)]$$

THE MODIFIED-GOODMAN DIAGRAM



$$\sigma_{a,allow} = \frac{46,000 \text{ psi}}{2.50} \rightarrow d = \sqrt[3]{\frac{293,395 \text{ in-lb}}{18,400 \text{ psi}}} = 2.51 \text{ in}$$

RE-DRAW GOODMAN LINE @ 2.5 in. dia. $\rightarrow S_e = 47,230 \text{ psi}$

$$\text{NOW } \sigma_{a,allow} = \frac{42,500 \text{ psi}}{2.50} \rightarrow d = \sqrt[3]{\frac{293,395 \text{ in-lb}}{17,000 \text{ psi}}} = 2.58 \text{ in}$$

$$\text{DOUBLE-CHECK } \sigma_{m,allow} = \frac{18,000 \text{ psi}}{2.50} \rightarrow d = \sqrt[3]{\frac{13 \cdot 70,640 \text{ in-lb}}{7,200 \text{ psi}}} = 2.57 \text{ in}$$

$d = 2\frac{5}{8} \text{ in}$