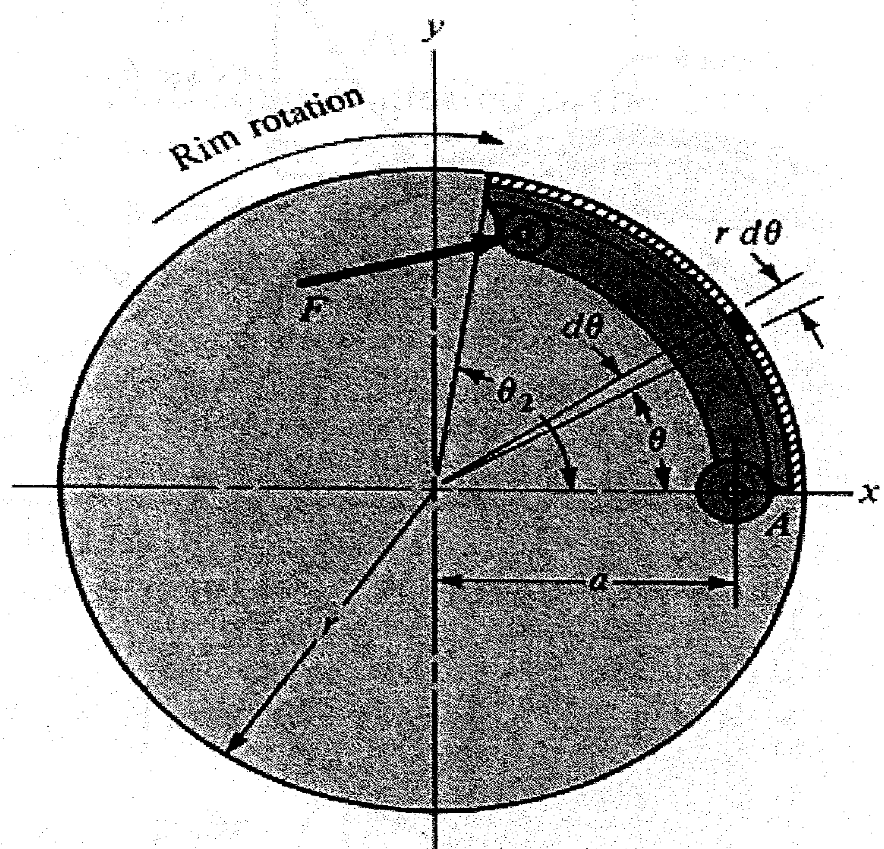
BRAKES ~ EXAMPLE PROBLEM
Reference: Shigley, 5th Edition
Problem 16.4

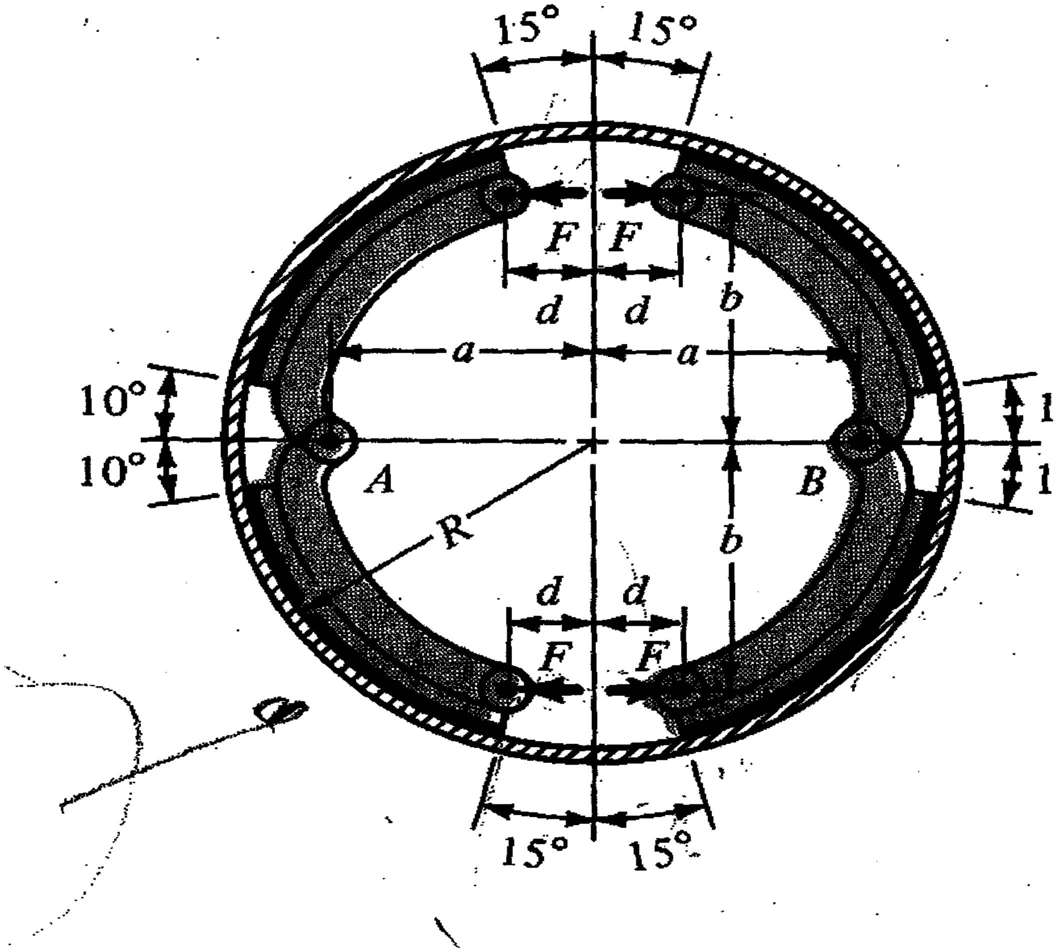
The figure below shows a 400-mm diameter brake drum with four (4) internally expanding shoes. Each of the hinge pins A and B supports a pair of shoes. The actuating mechanism is to be arranged to produce the same force F on each shoe. The face width of the shoes is 75-mm. The material used has a coefficient of friction of 0.24 and a maximum pressure of 1-mPa. Determine:

(a) Actuating force
(b) Braking capacity
(c) Noting the rotation may be in either direction, compute the hinge-pin reactions.

$$p_\theta = p_a \frac{\sin\theta}{\sin\theta_a}$$



Internal Friction Shoe



Dimensions are in millimeters and are:

a = 150
b = 165
d = 50

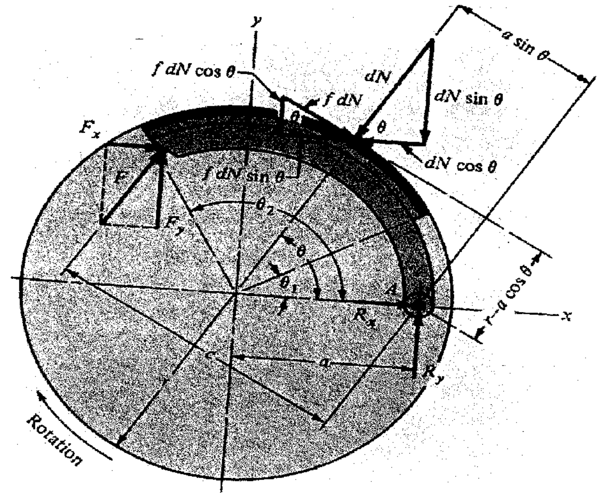
p = unit pressure acing upon an element of area of the frictional material located at an angle θ from the hinge pin.

p_a= maximum pressure located at an angle θ_a from the hinge pin.

p = max @ θ = 90-degrees.

If the ***toe angle,*** $\boldsymbol{\theta_2}$, is less than 90-degrees, maximum pressure will occur at the toe.

When $\theta = 0$, $p = 0$; material at the toe contributes little to the friction force.



Brake Shoe Forces

$$\boxed{M_f = \frac{f p_a b r}{\sin\theta_a}\int_{\theta_1}^{\theta_2} \sin(\theta)(r - a\cos\theta)\,d\theta}$$

= moment of frictional forces about pin.

$$\boxed{M_N = \frac{p_a b r a}{\sin\theta_a}\int_{\theta_1}^{\theta_2} \sin^2\theta\,d\theta}$$

= moment of normal forces about pin

$$\boxed{F = \frac{M_N - M_f}{c}}$$ ~ Actuating force

$$\boxed{M_N = M_f}$$ ~ self-locking

a must be s.t $M_N > M_f$ – for self-energizing.

Above derived from:

$$M_f = \int f N (r - a\cos\theta)\, d\theta$$

$$M_N = \int dN (a \sin\theta)\, d\theta$$

Torque applied by shoe:

$$T = \int f r\, dN = \frac{f p_a b r^2}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$

$$\boxed{T = \frac{f p_a b r^2 (\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}}$$

Pin reactions:

$$R_x = \int dN \cos\theta - \int f\, dN \sin\theta - F_x$$

$$= \frac{p_a b r}{\sin\theta_a} \left(\int_{\theta_1}^{\theta_2} \sin\theta \cos\theta\, d\theta \right.$$

$$\left. - f \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta \right) - F_x$$

Similarly:

$$R_y = \frac{p_a b r}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta + f \int_{\theta_1}^{\theta_2} \sin\theta \cos\theta\, d\theta$$

$$- F_y$$

<u>IF</u> direction reversed

$$F = \frac{M_N + M_f}{c}$$

$$A = \int_{\theta_1}^{\theta_2} \sin\theta \cos\theta \, d\theta = \frac{1}{2}\sin^2\theta \Big|_{\theta_1}^{\theta_2}$$

$$B = \int_{\theta_1}^{\theta_2} \sin^2\theta \, d\theta = \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]\Big|_{\theta_1}^{\theta_2}$$

:

$$R_x = \frac{p_a b r}{\sin\theta_a}(A - fB) - F_x$$

$$R_y = \frac{p_a b r}{\sin\theta_a}(B + fA) - F_y$$

:

$$R_x = \frac{p_a b r}{\sin\theta_a}(A + fB) - F_x$$

$$R_y = \frac{p_a b r}{\sin\theta_a}(B - fA) - F_y$$

Refence system ~ origin at center drum.

- Positive x-axis, thru hinge pin
- Positive y-axis, in direction of shoe.

Problem:

$$\theta_1 = 10^\circ$$

$$\theta_2 = 75^\circ$$

$$\therefore \theta_a = 75^\circ$$

Let $\alpha = \left[r\int_{\theta_1}^{\theta_2} \sin\theta \, d\theta - a\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta \, d\theta\right]$

(1) $\alpha = \left[r\int_{\theta_1}^{\theta_2} \sin\theta \, d\theta - aA\right]$

③ $\beta = \int_{\theta_1}^{\theta_2} \sin^2\theta \, d\theta = \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_{10}^{75}$
$= B$

$A = \int_{\theta_1}^{\theta_2} \sin\theta \cos\theta \, d\theta = \frac{1}{2}\sin^2\theta \Big|_{10}^{75}$

$\cancel{= \int_{10}^{75} \sin 2\theta}$

$A = 0.451$

$B = 0.527$

$\alpha = r\left[-\cos\theta\right]_{\theta_1}^{\theta_2} - aA$

$r = 200$ mm

$a = 150$ mm

$\alpha = 77.5$ mm

$$\boxed{M_f = \left(\frac{f p_a b r}{\sin\theta_a}\right)\alpha}$$

$$= \frac{(0.24)[1\times10^6 \frac{N}{m^2}](0.075\text{ m})(0.200\text{ m})}{\sin(75)} (0.0775)\text{ m}$$

$$\boxed{M_f = 290 \text{ N}\cdot\text{m}}$$

$$\boxed{M_N = \frac{p_a b r a}{\sin\theta_a} B}$$

$$M_N = \frac{(1\times10^6 \frac{N}{m^2})(.075\text{m})(.2\text{ m})(.15\text{ m})}{\sin(75)} (0.53)$$

$M_N = 1230$ N·m

$$\boxed{F = \frac{M_N - M_f}{c}}$$

$$= \frac{1230 - 290}{.165} \ \frac{N \cdot m}{m}$$

$$\boxed{F = \begin{cases} 5.7 \ kN \\ 5700 \ N \end{cases}}$$

For the primary shoe:

$$\boxed{T = \frac{f p_a b r^2 (\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}}$$

$$T = \frac{(.15 m)(1 \times 10^6 \frac{N}{m^2})(\cos 75 - \cos 10)(0.29)(.2)^2 m^2}{\sin 75}$$

$$\boxed{T = 541 \ N \cdot m}$$

For the secondary shoe:

$$\boxed{F = \frac{M_N + M_f}{c}} = 5700 \ N$$

$$\boxed{(F_{primary} = F_{secondary})}$$

$$M_N = \frac{1230}{10^6} p_a$$

$$M_f = \frac{289}{10^6} p_a$$

$$5700 = \frac{(1230/10^6) p_a + (289/10^6) p_a}{.165}$$

$$\boxed{p_a = 0.62 \times 10^6 \ Pa}$$

$T = 335\ N\cdot m$

$T_{TOTAL} = T_L + T_R$

$= (2)(541) + (2)(335)$

$$\boxed{\begin{aligned} T_{TOTAL} &= 1750\ N\cdot m \\ &= \text{Broking capacity} \end{aligned}}$$

$$\boxed{\begin{aligned} R_x &= -0.65\ kN \\ R_y &= 3.87\ kN \end{aligned}}$$ Primary

$$\boxed{\begin{aligned} R_x &= -0.14\ kN \\ R_y &= 4.02\ kN \end{aligned}}$$ Secondy