The following gearset problem is based upon problem #11-3,11-14, and 11-17 of Norton's text.

Pressure angle = 25-degrees
Number of teeth for the gear = 57, for the pinion 27.
The diametral pitch is 6
Quality level of the gears is 11.
Face width factor of 12 (12/P = F)

Gear material is A7-d nodular iron:
AGMA bending strength is 27 ksi
AGMA wear strength is 92 ksi
Brinell hardness is 230 ksi

Pinion material is a nitrided steel (4140) with the following strengths:
AGMA bending strength is 88 ksi
AGMA wear strength is 225 ksi
Hardness is 84.6N which corresponds to a BHN of approximately 1100.

The input torque to the pinion is 125 horse power and the angular velocity of the pinion is 1000-rpm.

Find:

(a) Circular pitch and pitch diameter for the gear and pinion.

(b) Determine the factor of safety and wear for the two gears based upon the following data. Also determine the angular velocity of the gear and the torque transmitted from the gear to its shaft.

$N_g := 57$ teeth $N_p := 27$ teeth

$P := 6$ $\frac{\text{teeth}}{\text{inch}}$

$d_{p.g} := \frac{N_g}{P}$ $d_{p.g} = 9.5$ inch

$d_{p.p} := d_{p.g} \cdot \frac{N_p}{N_g}$ $d_{p.p} = 4.5$ inch

$p := \frac{\pi}{P}$ $p = 0.524$ $\frac{\text{inch}}{\text{tooth}}$

$\omega_p := 1000$ rpm

$\omega_g := -\frac{N_p}{N_g} \cdot \omega_p$ $\omega_g = -473.684$ rpm

The next step in the solution process will be to construct a free body diagram of the pinion, determine the forces acting on the pinion. Then, repeat the process for the gear. This will be followed by determination of the wear (contact) and bending stresses for the two. The corresponding allowable strengths will be determined with subsequent calculation of the respective factors of safety for the two gears.

The torque applied to the pinion, assuming the pinion is the driving gear, can be determined by the following relationships:

$T_p = H/\omega_p$

$= \{H \text{ (hp)}[6600 \text{ (in-lb/sec)}]\} / \{\omega\text{(rpm)}(2\pi/60 \text{ (rad/sec/rpm)}\}$

$H := 125$ hp

$$T_p := \frac{H\cdot(6600)}{\left[\omega_p\cdot\left(2\cdot\frac{\pi}{60}\right)\right]} \qquad T_p = 7.88\cdot 10^3 \qquad in - lb_f$$

$$F_{32t} := \frac{T_p}{\left(\frac{d_{p.p}}{2}\right)} \qquad F_{32t} = 3.501\cdot 10^3 \qquad (pounds - force)$$

$$\phi := 25\cdot\frac{\pi}{180}$$

$$F_{32r} := F_{32t}\cdot\tan(\phi) \qquad F_{32r} = 1.633\cdot 10^3 \qquad lb_f$$

$$F_{23t} := F_{32t} \qquad F_{23r} := F_{32r}$$

$$T_{b3} := F_{23t}\cdot\left(\frac{d_{p.g}}{2}\right) \qquad T_{b3} = 1.663\cdot 10^4 \qquad in - lb_f$$

PINION - CONTACT STRESSES

The pinion contact stresses are calculated using the following equation:

$\sigma_c = C_p$ sqrt{[w_t/(FID)][C_aC_M/C_V]C_SC_f }

$\sigma_{c,all} = \sigma_c\ [(C_LC_H)/C_TC_R)]$

$N_{F,C} = \sigma_{C,ALL}/\sigma_C$

$F := \frac{12}{P} \qquad F = 2$ Face width, inches

$C_P := 2160$ $\sqrt{(psi)}$

$m_G := \frac{N_g}{N_p}$

$m_N := 1$ M_N = 1, spur gears

$I := \left(\frac{\cos(\phi)\cdot\sin(\phi)}{2\cdot m_N}\right)\cdot\left[\frac{m_G}{(m_G + 1)}\right]$ $I = 0.13$

$C_a := 1.0$

$V := \left(\frac{d_{p.g}}{2}\right)\cdot\omega_p\cdot\left[\frac{2\cdot(\pi)}{12}\right]$ $V = 2.487\cdot 10^3$ $\frac{ft}{min}$

From charts, for quality level of 11: $C_V := 0.9$

$C_M := 1.3$ $C_S := 1.0$ $C_f := 1.0$

$\sigma_c := C_P\cdot\sqrt{\left[\frac{F_{23t}}{(F\cdot I\cdot d_{p.p})}\right]\cdot\left[\frac{C_a\cdot(C_M)}{C_V}\right]\cdot C_S\cdot C_f}$

$\sigma_c := 142$ ksi

$C_L := 1.0$ Chart value

$A := 0.00698$ Eqn 11.26d, page 754 of text

$C_H := 1 + A\cdot(m_G - 1.0)$ Eqn #11.26a, page 754 of text.

$C_H = 1.008$

$C_R := 1.0$ Assumed R = 99%

$C_T := 1.0$

$S_c := 155$

$\sigma_{c.all} := \frac{C_L \cdot C_H}{(C_T \cdot C_R)} \cdot S_c$ $\sigma_{c.all} = 156.2$ psi

$N := \frac{\sigma_{c.all}}{\sigma_c}$ $N = 1.1$ Factor of Safety for Wear

For pinion bending, the equation is:

$\sigma_b = \{[W_t P/(FJ)][K_a K_M/K_V][K_S K_B K_I]\}$

$J := 0.46$ Table 11-13 for J based upon 25 pressure angle and number of teeth on pinion and gear.

$K_a := C_a$

$K_M := C_M$ $K_V := C_V$ $K_S := 1.0$ $K_B := 1.0$

$K_I := 1.0$

$$\sigma_b := \left[\frac{F_{32t} \cdot P}{(F \cdot J)}\right] \cdot \left(\frac{K_a \cdot K_M}{K_V}\right) \cdot K_S \cdot K_B \cdot \frac{K_I}{1000}$$

$\sigma_b = 33$ ksi

$K_L := 1.0$ From charts

$K_R := 1.0$

$S_t := 34$ AGMA bending strength, ksi

$K_T := 1.0$

$$\sigma_{t.all} := \frac{K_L}{\left(K_T \cdot K_R\right)} \cdot S_t \qquad \sigma_{t.all} = 34 \qquad \text{ksi}$$

$$N := \frac{\sigma_{t.all}}{\sigma_b} \qquad N = 1.031$$ Factor of safety in bending

The next step is to evaluate the gear using the same approach with the same data. Some of the modifying factors may change because we are now working with the gear instead of the pinion. And, because the materials have changed. For example, the pitch diameter of the gear will change as well as the value for J. Parameters such as the modifying factors for velocity, size, application, etc., will not change.

Only the equations and constants necessary to complete the gear portion of the evaluation are shown below.

$$\sigma_c := C_P \cdot \sqrt{\left[\frac{F_{23t}}{\left(F \cdot I \cdot d_{p.g}\right)}\right] \cdot \left[\frac{C_a \cdot \left(C_M\right)}{C_V}\right] \cdot C_S \cdot C_f}$$

$\sigma_c = 9.776 \cdot 10^4$ psi

$S_c := 92000$ psi

$$\sigma_{c.all} := \frac{C_L \cdot C_H}{\left(C_T \cdot C_R\right)} \cdot S_c \qquad \sigma_{c.all} = 9.271 \cdot 10^4 \qquad \text{psi}$$

$$N := \frac{\sigma_{c.all}}{\sigma_c} \qquad N = 0.948$$ Gear - factor of safety in wear - fails.

$J := 0.49$ Table 11-13 for J based upon 25^{0} pressure angle and number of teeth on pinion and gear.

$K_a := C_a$

$K_M := C_M$

$K_V := C_V$ $\quad K_S := 1.0$ $\quad K_B := 1.0$ $\quad K_I := 1.0$

$$\sigma_b := \left[\frac{F_{32t} \cdot P}{(F \cdot J)}\right] \cdot \left(\frac{K_a \cdot K_M}{K_V}\right) \cdot K_S \cdot K_B \cdot K_I \qquad \sigma_b = 3.096 \cdot 10^4 \quad \text{psi}$$

$K_L := 1.0$ From charts

$K_R := 1.0$

$S_t := 27000$ AGMA bending strength, psi

$K_T := 1.0$

$$\sigma_{t.all} := \frac{K_L}{\left(K_T \cdot K_R\right)} \cdot S_t \qquad \sigma_{t.all} = 2.7 \cdot 10^4 \quad \text{psi}$$

$$N := \frac{\sigma_{t.all}}{\sigma_b} \qquad N = 0.872$$ Gear factor of safety in bending.

The factors-of-safety in both bending and wear are less than 1, indicating a negative margine of safety. The next steps would be to increase the face width of the gears. For a second estimate try F = 2.5. Only the gear will be checked.

$F := 2.5$

$$\sigma_c := C_P \cdot \sqrt{\left[\frac{F_{23t}}{\left(F \cdot I \cdot d_{p.g}\right)}\right] \cdot \left[\frac{C_a \cdot \left(C_M\right)}{C_V}\right] \cdot C_S \cdot C_f}$$

$\sigma_c = 8.744 \cdot 10^4$ psi

$S_c := 92000$ psi

$$\sigma_{c.all} := \frac{C_L \cdot C_H}{(C_T \cdot C_R)} \cdot S_c \qquad \sigma_{c.all} = 9.271 \cdot 10^4 \quad \text{psi}$$

$$N := \frac{\sigma_{c.all}}{\sigma_c} \qquad N = 1.06$$

Gear - factor of safety in wear - ok.

$J := 0.49$

$K_a := C_a$

Table 11-13 for J based upon 25° pressure angle and number of teeth on pinion and gear.

$K_M := C_M$

$K_V := C_V \qquad K_S := 1.0 \qquad K_B := 1.0 \qquad K_I := 1.0$

$$\sigma_b := \left[\frac{F_{32t} \cdot P}{(F \cdot J)}\right] \cdot \left(\frac{K_a \cdot K_M}{K_V}\right) \cdot K_S \cdot K_B \cdot K_I \qquad \sigma_b = 2.477 \cdot 10^4 \quad \text{psi}$$

$K_L := 1.0$

$K_R := 1.0$

From charts

$S_t := 27000$

AGMA bending strength, psi

$K_T := 1.0$

$$\sigma_{t.all} := \frac{K_L}{(K_T \cdot K_R)} \cdot S_t \qquad \sigma_{t.all} = 2.7 \cdot 10^4 \quad \text{psi}$$

$$N := \frac{\sigma_{t.all}}{\sigma_b} \qquad N = 1.09$$

Gear factor of safety in bending, ok.