RELIABILITY MODEL FOR BIOLOGICAL OBJECTS [1]

Vladimir Rykov (E-mail: rykov@rykov1.ins.ru)
Dept. of Appl. Math. and Comp. Modeling, Russian State University of Oil&Gas
Leninskiy prosp., 65, Moscow 117917, Russia
Boyan Dimitrov (E-mail: bdimitro@kettering.edu),
David Green Jr. (E-mail: dgreen@kettering.edu),
Peter Stanchev (E-mail: pstanche@kettering.edu)
Dept. of Science and Mathematics, Kettering University
1700 West Third Ave., Flint, Mi 48504, USA,

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In this paper wu use some ideas for analysis of reliability of technical systems to propose similar tools for modeling and investigation of the problem of biological objects longevity, aging, and degradation. The main characteristic for biological objects is their survival function. We focus on evaluation of this function. We deal with the model of hierarchical structure, which is described in the framework of a multi-dimensional Markov process with specific transition intensities.

1 Introduction and Motivation

In terms of reliability, most of the biological objects with sufficiently high organization are complex hierarchical controllable systems. Even the lowest level living organisms possess tools for viability control, for defense and treatment. The organism is a system consisting of several subsystems. Each subsystem can be divided into subsystems of lower level etc. up to the smallest entities, which we will refer to as units. A failure of any unit does not usually lead to immediate failure of the respective subsystem to which this unit belongs, neither can this cause a failure of subsystems of higher level. Moreover, most units also do not fail immediately, but usually pass through several stages of loss of their efficiency.

The neuron system play a role of controlling system, and for a living body, for instance, it possesses high reliability. This means that biological objects can be treated as a complex hierarchical controllable fault tolerance reliability systems [1] - [4].

In this paper we propose a general mathematical model for the description, evaluation and control of most common reliability characteristics of complex hierarchical systems with multi-state fault tolerance units under various repair policies. The main characteristic for biological objects is their survival function. Thus, in this paper we focus on evaluation of this function. The model is described in the framework of a multi-dimensional Markov process with specific transition intensity structure.

The specifics of any particular system is represented in the form of a special structure of mutually interacting components and subsystems, with the sets of its normal functioning, dangerous and failure states, specified in some transition graphs and/or a failure tree. Since these are individual characteristics for any concrete system, we touch them in our model only indirectly. We merely assume that the user can determine the set of the system states and detach its appropriate subsets of normal functioning, dangerous and failure states, and also assign appropriate specifics for the system functioning. Moreover, because the main problem is the particular model description with its parameters specialization we propose some general algorithm for construction of a large scope of feasible models, and for supplying it with parameters needed for further analysis. Due to the complexity of the problem, most of our solutions have an algorithmic form. These algorithms are included in the computer tools mentioned above for the system modeling.

Our model also allows the use of linguistic data, the most common data processing in biological objects description. Linguistic data are converted to fuzzy values, which are entered in the model. The model results can also be converted back in linguistic form.

In the study we deal mainly with homogeneous in time models. In practice, the durability, aging, and degradation studies one needs to model the failures and treatments intensities as a time dependent function. An appropriate generalization of the model is possible, but its analysis requires specific information and a wider use of deep numerical analysis and computer tools.

Some simple example illustrate our approach.

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2 A General Model

Consider a complex hierarchical multi-component system which is controlled and managed by a high reliable system of control (SoC), as shown in Fig 1.

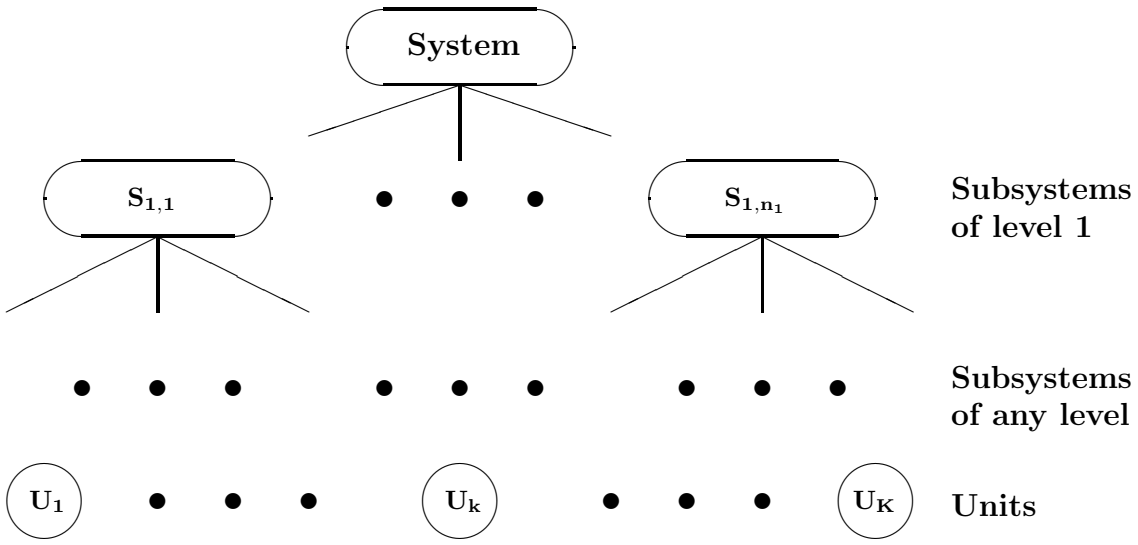

Fig. 1. A complex multi-level hierarchical system.

In biology linguistic values are very often used. For instance "more or less enlarged" or "too much enlarged" do not have an exact mapping the scale of real numbers. Rather we could map each linguistic value to a fuzzy number. A fuzzy number is a number which is "around" a real number. A fuzzy number is represented by its membership function $h(x)$ defined with domain some real interval and range within [0, 1], located at the most likely value of the variable it represents. Its maximum value is equal to 1. For example the value "suspected not normal" for liver enlargement could mean that volume of the liver is between 85% and 130% of its normal size, and that the value is most probably around 110%. Gathering opinion from several experts on what is "suspected not normal" for liver enlargement means, we could calculate a fuzzy number corresponding to "suspected not normal" and have a graphical representation of the linguistic concept. It is similar to the graph of a uni-modal function $h(x)$ defined over an interval which contains [85,130] with a maximum value equal to 1, located just above the point $x_0 = 110$, and descending when x gets away from its most likely value 110.

In our work we convert linguistic values into fuzzy numbers (functions), work with them performing all the specified operations, convert after that the result as new linguistic values and interpret it correspondingly.

Assume that the system is constructed from blocks and branches of several, L, levels. Each block with its branches is followed by other blocks that may form a hierarchical subsystem of the same type as the main one. The blocks of the last (lowest) level will be referred as units (elements) and may be subjected to gradual (internal) failures of its own type. We will denote by L the maximal level of units. It is not necessary that any unit belongs to this level. Different level units are possible. The reliability of each unit is partially controllable.

The failures in the systems of this type arise as a result of stress accumulation of the lowest level elements which pass several stages before full failure. These faults lead to the efficiency of the system decreasing but do not lead to the full failure of the system. The system of control (SoC) fixes these fault stages of elements and gives a signal about the system "state of health" decreasing.

Accordingly, to these signals, appropriate mechanisms of self-regulation are "switched in", and the system is self regenerated if the process disturbing is not too deep. In the last case some outside acting is needed. It is supposed that these action being applied at the time and in needed quality and quantity turn the system after some time to the normal functioning state.

In another case the delay with maintenance of the system leads to the system degradation and as a result it leads to the full failure of the system.

In order to specify the description of its behavior we introduce the vector index $i = (i_1, i_2, \ldots, i_{L_i})$ which determines each unit of the system as belonging to an appropriate chain of blocks at any level. Denote by $\mathcal{I}$ the set of these indices (and appropriate units). Then the state space $\mathcal{E}$ of the system can be represented as a set of vectors $\mathbf{x} = \{x_i : \ i \in \mathcal{I})\}$, where for any $i \in \mathcal{I}$ the integer x_i represents the state of the i-th unit in sense of its reliability. It can take different values, depending on its type, $x_i \in \{0, 1, \ldots m_i\}$, where the exhausted

level of i-th unit is denoted by m_i. Notice that these numbers have no specific physical sense, but indicate only a possible level of gradual failure of the i-th unit. The value of $x_i = m_i$ means the full failure of i-th unit. Denote also by $\mathcal{N}$ the set of all normal system states, $\mathcal{D}$ the set of defective states, and $\mathcal{F}$ the set of the full system breakdown states. Moreover it is supposed that these subsets contain "boundary" sub-subsets $\Gamma_{\mathcal{ND}}$, $\Gamma_{\mathcal{NF}}$ and $\Gamma_{\mathcal{DF}}$, such that the transition to the states in $\mathcal{D}$ and $\mathcal{F}$ is possible only from the states from these subsets.

To model the system as functioning according to a finite state Markov process we assume that the times of transition from one gradual level to another, as well as the times to repair a failed units have exponential distributions. The respective parameters may depend on the type of the unit $i \in \mathcal{I}$ and also on the entire system state $\mathbf{x}$. These assumptions allow us to model the reliability of such a system by a multi-dimensional Markov process

$$\mathbf{X} = \{X_i(t) : \ i \in \mathcal{I}, \ t \geq 0\},$$

with set of states $\mathcal{E}$, which should be precise for any specific system.

Additional assumption concerns the structure of transition intensities of such a process. The specifics of the reliability models make it reasonable to suppose that the process can jump only to neighboring states (in the case that gradual failure arises), and some function f determines the states to which the process goes in the case of a completed treatment. The treatment function f can be given in different ways. The cases, where as a treatment resulted in which the whole system is renovated, and only a unit is renovated were considered in [3], [4]. Other treatment policies are also possible. For example, some subsystem of a given level could be renovated as a treatment result.

This means that the transition intensities have the following form

$$a(\mathbf{x},\mathbf{y}) = \begin{cases} \alpha_i(\mathbf{x}) & \text{for } \mathbf{y} = \mathbf{x} + \mathbf{e}_i, \ \mathbf{x},\mathbf{y} \in \mathcal{N}, \\ \lambda_i(\mathbf{x}) & \text{for } \mathbf{y} = \mathbf{x} + \mathbf{e}_i, \ \mathbf{x} \in \Gamma_{\mathcal{N}}, \mathbf{y} \in \mathcal{D}, \\ \beta_i(\mathbf{x}) & \text{for } \mathbf{y} = \mathbf{x} + \mathbf{e}_i, \ \mathbf{x},\mathbf{y} \in \mathcal{D}, \\ \mu_i(\mathbf{x}) & \text{for } \mathbf{y} = f(\mathbf{x}), \ \mathbf{x} \in \mathcal{D}, \\ \gamma_i(\mathbf{x}) & \text{for } \mathbf{y} = \mathbf{x} + \mathbf{e}_i, \ \mathbf{x} \in \Gamma_{\mathcal{N}}, \mathbf{y} \in \mathcal{F}, \\ \nu_i(\mathbf{x}) & \text{for } \mathbf{y} = \mathbf{x} + \mathbf{e}_i, \ \mathbf{x} \in \Gamma_{\mathcal{N}}, \mathbf{y} \in \mathcal{F}. \end{cases} \tag{1}$$

Here and later notation $\mathbf{e}_k$ means a unit vector with 1 at the k-th position and zeros elsewhere, while $\alpha(\mathbf{x})$, $\beta(\mathbf{x})$, $\gamma(\mathbf{x})$, $\lambda(\mathbf{x})$, $\mu(\mathbf{x})$ and $\nu(\mathbf{x})$ denote the sums of appropriate intensities over all admissible in the state $\mathbf{x}$ indices.

The graph of transition with appropriate intensities for typical states is shown in Fig. 2.

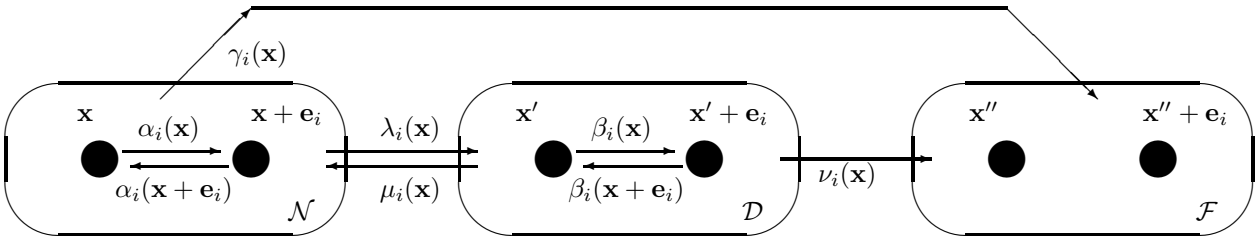

Fig. 2. Transition graph the Model.

3 A General Model Investigation

The Kolmogorov's system of differential equations for the time dependent probabilities of the process with transition intensities (1) and transition graph given in Fig. 2 has the form

$$\begin{aligned} \frac{d\pi(\mathbf{0};t)}{dt} &= -\lambda(\mathbf{0})\pi(\mathbf{0};t) + \sum_{i\in\mathcal{I}} \mu_i(\mathbf{e}_i)\pi(\mathbf{e}_i;t) + \sum_{\mathbf{y}\in f^{-1}(\mathbf{0})\subset\mathcal{D}} \mu(\mathbf{y})\pi(\mathbf{y};t), \\ \frac{d\pi(\mathbf{x};t)}{dt} &= -(\alpha(\mathbf{x}) + \lambda(\mathbf{x}) + \gamma(\mathbf{x}))\pi(\mathbf{x};t) + \sum_{\mathbf{x}-\mathbf{e}_i\in\mathcal{N}:x_i\neq 0} \alpha_i(\mathbf{x}-\mathbf{e}_i)\pi(\mathbf{x}-\mathbf{e}_i;t) + \\ &+ \sum_{\mathbf{y}\in f^{-1}(\mathbf{x})\subset\mathcal{D}} \sum_{\mathbf{y}\in\mathcal{D}} \mu_i(\mathbf{y})\pi(\mathbf{y};t), \qquad \mathbf{x}\in\mathcal{N} \end{aligned}$$

$$
\begin{aligned}
\frac{d\pi(\mathbf{x};t)}{dt} &= -(\beta(\mathbf{x})+\mu(\mathbf{x})+\nu(\mathbf{x}))\pi(\mathbf{x};t) + \sum_{\mathbf{x}-\mathbf{e}_i\in\Gamma_{\mathcal{ND}}:x_i\neq 0}\lambda_i(\mathbf{x}-\mathbf{e}_i)\pi(\mathbf{x}-\mathbf{e}_i;t) + \\
&+ \sum_{\mathbf{x}-\mathbf{e}_i\in\mathcal{D}:x_i\neq 0}\beta_i(\mathbf{x}-\mathbf{e}_i)\pi(\mathbf{x}-\mathbf{e}_i;t), \qquad \mathbf{x}\in\mathcal{D}, \\
\frac{d\pi(\mathbf{x};t)}{dt} &= \sum_{\mathbf{x}-\mathbf{e}_i\in\Gamma_{\mathcal{NF}}}\gamma(\mathbf{x}-\mathbf{e}_i)\pi(\mathbf{x}-\mathbf{e}_i;t) + \sum_{\mathbf{x}-\mathbf{e}_i\in\Gamma_{\mathcal{DF}}}\nu(\mathbf{x}-\mathbf{e}_i)\pi(\mathbf{x}-\mathbf{e}_i;t), \quad \mathbf{x}\in\mathcal{F}. \qquad (2)
\end{aligned}
$$

The system solution can be represented in terms of the Laplace transforms (LT)

$$\tilde{\pi}(\mathbf{x};s)=\int_0^\infty e^{-st}\pi(\mathbf{x};t)dt.$$

These functions make it possible to calculate the moments of the failure time, and perform its asymptotic analysis for the large t. Passing into the system (11) to the Laplace transform with the initial condition $\pi(\mathbf{0};0)=1$ one finds that

$$
\begin{aligned}
(s+\lambda(\mathbf{0}))\tilde{\pi}(\mathbf{0};s)-1 &= \sum_{i\in\mathcal{I}}\mu_i(\mathbf{e}_i)\tilde{\pi}(\mathbf{e}_i;s) + \sum_{\mathbf{y}\in f^{-1}(\mathbf{0})\subset\mathcal{D}}\mu(\mathbf{y})\tilde{\pi}(\mathbf{y};s), \\
(s+\alpha(\mathbf{x})+\lambda(\mathbf{x})+\gamma(\mathbf{x}))\tilde{\pi}(\mathbf{x};s) &= \sum_{\mathbf{x}-\mathbf{e}_i\in\mathcal{N}:x_i\neq 0}\alpha_i(\mathbf{x}-\mathbf{e}_i)\tilde{\pi}(\mathbf{x}-\mathbf{e}_i;s) + \\
&+ \sum_{\mathbf{y}\in f^{-1}(\mathbf{x})\subset\mathcal{D}}\sum_{\mathbf{y}\in\mathcal{D}}\mu_i(\mathbf{y})\tilde{\pi}(\mathbf{y};s), \qquad \mathbf{x}\in\mathcal{N} \\
(s+\beta(\mathbf{x})+\mu(\mathbf{x})+\nu(\mathbf{x}))\tilde{\pi}(\mathbf{x};s) &= \sum_{\mathbf{x}-\mathbf{e}_i\in\Gamma_{\mathcal{ND}}:x_i\neq 0}\lambda_i(\mathbf{x}-\mathbf{e}_i)\tilde{\pi}(\mathbf{x}-\mathbf{e}_i;s) + \\
&+ \sum_{\mathbf{x}-\mathbf{e}_i\in\mathcal{D}:x_i\neq 0}\beta_i(\mathbf{x}-\mathbf{e}_i)\tilde{\pi}(\mathbf{x}-\mathbf{e}_i;s), \qquad \mathbf{x}\in\mathcal{D}, \\
s\tilde{\pi}(\mathbf{x};s) &= \sum_{\mathbf{x}-\mathbf{e}_i\in\Gamma_{\mathcal{NF}}}\gamma(\mathbf{x}-\mathbf{e}_i)\tilde{\pi}(\mathbf{x}-\mathbf{e}_i;t) + \\
&+ \sum_{\mathbf{x}-\mathbf{e}_i\in\Gamma_{\mathcal{DF}}}\nu(\mathbf{x}-\mathbf{e}_i)\tilde{\pi}(\mathbf{x}-\mathbf{e}_i;t), \qquad \mathbf{x}\in\mathcal{F}. \qquad (3)
\end{aligned}
$$

For two special cases when the "treatment" of the system leads to the full system renovation and for the renovation only one unit the closed form and algorithmic solutions of this system have been done in [2] and [3]. In the general case the solution could be find by the numerical methods.

Instead of this we focus on the asymptotic behavior of the system survival function. Taking into account that the transition intensities inside subsets $\mathcal{N}$, $\mathcal{D}$ and $\mathcal{F}$ have the same order and the transition intensities between these subsets have much smaller order we use V. Korolyuk's method of asymptotic enlarging of the system states (see [5]) to reduce the model to a simple model, where all normal, dangerous and failure states are brought together. Such a model is represented in the next section.

4 An Enlarge State Model

Consider the system after appropriate states of a general model enlarging. An appropriate process can pass through only three states: the normal functioning state N, the dangerous (pre-failure) state D and the failure state F. In spite of the simplicity of the model, remarkable results are obtained concerning the degradation and aging of the system. The marked transition graph for the process is shown in Fig. 3. The transition intensities are also included in the figure.

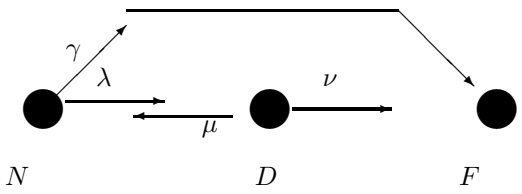

Fig. 3. Transition graph for Enlarge State Model.

According to this transition graph the Kolmogorov's system of differential equations for time-dependent state probabilities has the form

$$\begin{aligned}
\frac{d\pi_N(t)}{dt} &= -(\lambda+\gamma)\pi_N(t) + \mu\pi_D(t), \\
\frac{d\pi_D(t)}{dt} &= -(\mu+\nu)\pi_D(t) + \lambda\pi_N(t), \\
\frac{d\pi_F(t)}{dt} &= \gamma\pi_N(t) + \nu\pi_D(t).
\end{aligned} \tag{4}$$

The survival (reliability) function for the system is given by

$$S(t) = 1 - \pi_F(t) = 1 - \int_0^t \left[\gamma\pi_N(u) + \nu\pi_D(u)\right] du, \tag{5}$$

where the functions $\pi_N(t)$ and $\pi_D(t)$ are the solution of the first two equations of system (4). We find this solution in terms of their Laplace transforms $\tilde{\pi}_N(s)$ and $\tilde{\pi}_D(s)$. Appropriate algebraic system in terms of the LS-transforms for two first equations of the system (4) with the initial condition $\pi_N(0) = 1$ takes the form

$$\begin{aligned}
(s+\lambda+\gamma)\tilde{\pi}_N(s) - \mu\tilde{\pi}_D(s) &= 1, \\
-\lambda\tilde{\pi}_N(s) + (s+\mu+\nu)\tilde{\pi}_D(s) &= 0.
\end{aligned} \tag{6}$$

Its solution can be represented in the terms of roots of the characteristic equation

$$\Delta = s^2 + (\lambda+\mu+\nu+\gamma)s + \lambda\,\nu + \gamma\,(\mu+\nu) = 0.$$

To simplify this equation and its roots we use the substitution

$$p = \lambda+\mu+\nu+\gamma, \qquad q = \lambda\,\nu + \gamma\,(\mu+\nu).$$

With this notation the characteristic equation takes the form

$$\Delta = s^2 + ps + q = 0.$$

Its roots s_1 and s_2 (it is assumed that $s_2 < s_1 < 0$) are given by the formulae

$$s_{1,2} = -\frac{p}{2}\left(1 \mp \sqrt{1 - \frac{4q}{p^2}}\right). \tag{7}$$

Omitting some simple algebra we give the solution of the system (6) for LS-transforms

$$\begin{aligned}
\tilde{\pi}_N(s) &= \frac{1}{s_1 - s_2}\left(\frac{\mu+\nu+s_1}{s-s_1} - \frac{\mu+\nu+s_2}{s-s_2}\right), \\
\tilde{\pi}_D(s) &= \frac{\lambda}{s_1 - s_2}\left(\frac{1}{s-s_1} - \frac{1}{s-s_2}\right).
\end{aligned}$$

Using inverse transforms one can find

$$\begin{aligned}
\pi_N(t) &= \frac{\mu+\nu+s_1}{s_1-s_2}e^{s_1 t} - \frac{\mu+\nu+s_2}{s_1-s_2}e^{s_2 t}, \\
\pi_D(t) &= \frac{\lambda}{s_1-s_2}\left(e^{s_1 t} - e^{s_2 t}\right).
\end{aligned} \tag{8}$$

Thus, for the survival function (5) after integration of the last of equations (4) and some transformations we get

$$\begin{aligned}
S(t) &= 1 - \pi_F(t) = \frac{q+\gamma\,s_2}{(s_1-s_2)\,s_2}e^{s_2 t} - \frac{q+\gamma\,s_1}{(s_1-s_2)\,s_1}e^{s_1 t} \\
&= e^{s_1 t}\left(1 - \frac{\gamma+s_1}{s_1-s_2}\left(1 - e^{s_2-s_1}\right)\right)
\end{aligned} \tag{9}$$

To explain the longevity, aging and degradation problem let us consider the asymptotic behavior of this function under the condition

$$\frac{q}{p^2} = \epsilon, \quad as \quad \epsilon \to 0.$$

From the relation (7) one can find

$$s_1 \approx -p\epsilon, \qquad s_2 \approx -p(1-\epsilon).$$

Therefore for the survival function (9) we have

$$S(t) = e^{-p\epsilon t}\left(1 - \frac{\gamma - p\epsilon}{p(1-2\epsilon)}\left(1 - e^{-p\,(1-2\epsilon t}\right)\right) \tag{10}$$

The last expression shows that the survival function and therefore mean life time mainly depend on ratio $\epsilon = q/p^2$ and increase with this ratio decreasing.

To investigate the survival function in more details depending on different parameters behavior we assume that the intensity of "instantaneous" failure is smaller concerning with the intensity of "gradual" failure, while the latter is much more smaller then the treatment rate in the dangerous state, and this one is comparable with (has the same order as) transition intensity from dangerous to failure state, i.e. we put

$$\mu >> \lambda >> \gamma, \qquad \nu \approx \mu.$$

Dividing all the parameters over μ and scaling time in such a way that $\mu = 1$, we put

$$\mu = 1,\ \lambda = \epsilon,\ \gamma = \sigma\epsilon,\ \nu = c.$$

For these parameters one can evaluate that

$$\frac{q}{p^2} \approx \frac{c + \sigma(1+c)}{(1+c)^2}\epsilon - 2\frac{(c+\sigma(1+c))(1+\sigma)}{(1+c)^3}\epsilon^2,$$

and therefore the above approximation could be applied with appropriate coefficients.

5 Examples

As an Example we consider a simple general model with transition graph as on fig. 2 with 5 states in each set $\mathcal{N}$, $\mathcal{D}$, and $\mathcal{F}$. A birth and death process runs inside states $\mathcal{N}$ and $\mathcal{D}$ and transition intensities have different order. We use double indexation in the subscripts $\pi_{i,j}$ for immediate understanding. Moreover, $i = 0$ for all states in $\mathcal{N}$, $i = 1$ for all states in $\mathcal{D}$, and $i = 2$ for all states in the absorbing set $\mathcal{F}$, and $j = 1, 2, 3, 4, 5$. The system of differential equations for the state probabilities is as follow:

$$\begin{aligned}
\frac{d\pi_{0,1}(t)}{dt} &= -(\alpha_{1,2} + \gamma_1 + \lambda_1)\pi_{0,1}(t) + \alpha_{2,1}\pi_{0,2}(t) + \mu_1\pi_{1,1}(t),\\
\frac{d\pi_{0,j}(t)}{dt} &= -(\alpha_{j,j-1} + \alpha_{j,j+1} + \gamma_j + \lambda_j)\pi_{0,j}(t) + \alpha_{j-1,j}\pi_{0,j-1}(t) + \alpha_{j+1,j}\pi_{0,j+1} + \mu_j\pi_{1,j}(t), \quad j = 2,3,4\\
\frac{d\pi_{0,5}(t)}{dt} &= -(\alpha_{5,4} + \gamma_5 + \lambda_5)\pi_{0,5}(t) + \alpha_{4,5}\pi_{0,4}(t) + \mu_5\pi_{1,5}(t),\\
\frac{d\pi_{1,1}(t)}{dt} &= -(\beta_{1,2} + \nu_1 + \mu_1)\pi_{1,1}(t) + \beta_{2,1}\pi_{1,2}(t) + \lambda_1\pi_{0,1}(t),\\
\frac{d\pi_{1,j}(t)}{dt} &= -(\beta_{j,j-1} + \beta_{j,j+1} + \nu_j + \mu_j)\pi_{1,j}(t) + \beta_{j-1,j}\pi_{1,j-1}(t) + \beta_{j+1,j}\pi_{1,j+1} + \lambda_j\pi_{0,j}(t), \quad j = 2,3,4\\
\frac{d\pi_{1,5}(t)}{dt} &= -(\beta_{5,4} + \nu_5 + \mu_5)\pi_{1,5}(t) + \beta_{4,5}\pi_{1,4}(t) + \lambda_5\pi_{0,5}(t),\\
\frac{d\pi_{2,j}(t)}{dt} &= \gamma_j\pi_{0,j}(t) + \nu_j\pi_{1,j}(t), \quad j = 1,2,3,4,5.
\end{aligned} \tag{11}$$

We use MATLAB to solve the system. The survival function is calculated as

$$S_1(x) = \sum_{j=1}^{5}[\pi_{0,j}(t) + \pi_{1,j}(t)] \tag{12}$$

The numerical values given to the parameters are chosen as here. The $\alpha_{i,j}$'s are about 10 times μ_j's, about 100 times λ_i's, and about 1000 times γ_j's. We keep the proportions $\beta_{i,j} = .9\alpha_{i,j}$, and $\nu_j = .3\mu_j$, and no big deviations from state to state. Hence, the parameters for our example are:

$$\begin{aligned}
&\alpha_{1,2} = 10;\ \beta_{1,2} = 9,\ \mu_1 = 1,\ \lambda_1 = 0.1,\ \gamma_1 = 0.01,\ \text{and}\ \ \nu_1 = .3;\\
&\alpha_{j,j-1} = 9.5;\ \beta_{j,j-1} = 8.55,\ \alpha_{j,j+1} = 10.5;\ \beta_{j,j+1} = 9.45,\\
&\mu_j = .95,\ \lambda_j = 0.095,\ \gamma_j = 0.0095,\ \text{and}\ \ \nu_j = .28, \quad j = 2,3,4;\\
&\alpha_{5,4} = 10.5;\ \beta_{5,4} = 9.45,\ \mu_5 = 1.05,\ \lambda_5 = 0.105,\ \gamma_5 = 0.00105,\ \text{and}\ \ \nu_5 = .32;
\end{aligned}$$

The survival function of the above system is compared to the survival function (**??**) of the aggregated model described in Section 4. We chose as numerical values the average (arithmetic mean) of the intensities outgoing from each state set. Thus, we have

$$\lambda = \frac{1}{5}(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5) = .098; \quad \mu = .98, \quad \gamma = .0098, \quad \nu = .2812,$$

Substitute it in (**??**) and obtain

$$S_2(x) =$$

To compare the results for general model with analogous for the Enlarge State model we also consider the calculation of the survival function $S(t)$ accordingly to exact formulae (9) and approximate formulae (10) for the following value of parameters

$$\mu = 1,\ \nu = 0.5\ \lambda = 0.1,\ \gamma = 0.01.$$

For these values of parameters the exact formulae (9) gives

$$S(t) = e^{-0.0402t}(1.0196 - 0.0196e^{-1.5295t})$$

and the approximate formulae (10) gives

$$S(t) \approx e^{-0.0407t}(1.0197 - 0.0197e^{-1.5295t}).$$

For this case $\epsilon = qp^{-2} \approx 0.025$ and one can see that the approximate formulae gives enough good result

6 Conclusions

Some general model for degradation and survival of biological objects is proposed. For the case of high reliable object an approximation for survival function is given. An example shows the quality of approximation.

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