The integral

$$\int e^{-x^2}\,dx$$

arises frequently in analyses of many mechanical, electrical, chemical, and biological processes.

To 15 digits, the abscissas and weights for the 4–point and 5–point Gauss quadratures are

4–Point Gauss Quadrature

t_i	w_i
−0.86113 63115 94053	0.34785 48451 37454
−0.33998 10435 84856	0.65214 51548 62546
0.33998 10435 84856	0.65214 51548 62546
0.86113 63115 94053	0.34785 48451 37454

5–Point Gauss Quadrature

t_i	w_i
−0.90617 98459 38664	0.23692 68850 56189
−0.53846 93101 05683	0.47862 86704 99366
0.00000 00000 00000	0.56888 88888 88889
0.53846 93101 05683	0.47862 86704 99366
0.90617 98459 38664	0.23692 68850 56189

1. Approximate the integral

$$\int_{-1}^{4} e^{-x^2}\,dx$$

using 4–point Gauss quadrature. Answer: 1.6428 6173 7419.

2. Approximate the integral

$$\int_{-1}^{4} e^{-x^2}\,dx$$

using 5–point Gauss quadrature. Answer: 1.6485 3667 1769.

3. A 30–point Gauss quadrature will exactly integrate (without error) all polynomials up to what degree?

4. **True / False:** Gauss quadrature is used only to integrate polynomials.