MATH-305 Numeric Differentiation

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
f	1.54308	1.66852	1.81066	1.97091	2.15090	2.35241	2.57746	2.82832	3.10747

See these formulas.

1. Use the above data to approximate $f'(1.4)$ using the 3–point central difference formula with

   (a) $h = 0.1$ answer: 1.9075

   (b) $h = 0.2$ answer: 1.9170

   (c) Richardson extrapolation answer: $1.904\bar{3}$

   Use the correction term to **estimate** the error of (a) in permille. answer: $-1.66‰$

2. Use the above data to approximate $f'(1.4)$ using the 3–point forward difference formula with

   (a) $h = 0.1$ answer: 1.8974 000

   (b) $h = 0.2$ answer: 1.8741 750

   (c) Richardson extrapolation answer: $1.9051\ 41\bar{6}$

   Use the correction term to **estimate** the error of (a) in percent. answer: 0.408%

3. Use the above data to approximate $f'(1.4)$ using the 5–point central difference formula with

   (a) $h = 0.1$ answer: $1.904\bar{3}$

   (b) $h = 0.2$ answer: $1.9041\ 708\bar{3}$

   (c) Richardson extrapolation answer: $1.9043\ 441\bar{6}$

   Use the correction term to **estimate** the error of (a) in parts per million. answer: 5.689 ppm

4. Use the above data to approximate $f''(1.4)$ using the 5–point central difference formula with

   (a) $h = 0.1$ answer: 2.1500 0000

   (b) $h = 0.2$ answer: $2.1507\ 708\bar{3}$

   (c) Richardson extrapolation answer: $2.1499\ 486\bar{1}$

   Use the correction term to **estimate** the error of (a) in permyriad. answer: $-0.239‱$

5. Use the above data to approximate $f''(1.8)$ using the 5–point backward difference formula with

   (a) $h = 0.1$ answer: 3.0962 5000

   (b) $h = 0.2$ answer: $3.0940\ 208\bar{3}$

   (c) Richardson extrapolation answer: 3.0965 6845

   Use the correction term to **estimate** the error of (a) in permyriad. answer: $1.0285‱$

Use Newton-Gregory interpolating polynomials to derive the following 4–point difference formulas for $f'(x_0)$ and $f'(x_1)$.

$$f'(x_0) = \frac{-11f_0 + 18f_1 - 9f_2 + 2f_3}{6h}$$

$$f'(x_1) = \frac{-2f_0 - 3f_1 + 6f_2 - f_3}{6h}$$