Use Laplace transforms to solve the IVP

$$y'' - 6\,y' + 13\,y \;=\; 0\,, \tag{1}$$

$$y(0) \;=\; 0\,, \qquad y'(0) \;=\; -3 \tag{2}$$

1. **Take $\mathcal{L}$ of ODE (1):**

$$\mathcal{L}\{y''\} - 6\,\mathcal{L}\{y'\} + 13\,\mathcal{L}\{y\} \;=\; \mathcal{L}\{0\}\,, \tag{3}$$

where

$$\begin{aligned}
\mathcal{L}\{y\} &= Y\,, \\
\mathcal{L}\{y'\} &= s\,Y - y(0) \;=\; s\,Y - 0 \;=\; s\,Y\,, \\
\mathcal{L}\{y''\} &= s^2\,Y - s\,y(0) - y'(0) \;=\; s^2\,Y - s(0) - (-3) \;=\; s^2\,Y + 3\,, \\
\mathcal{L}\{0\} &= 0\,.
\end{aligned}$$

Substitute these into Eq. (3) to obtain

$$s^2\,Y + 3 - 6\,s\,Y + 13Y \;=\; 0\,.$$

2. **Solve for Y:**

$$(s^2 - 6\,s + 13)\,Y \;=\; -3\,,$$

so

$$\begin{aligned}
Y &= \frac{-3}{s^2 - 6\,s + 13} \\
&= \frac{-3}{s^2 - 6\,s + 9 - 9 + 13} \qquad \text{complete the square} \\
&= \frac{-3}{(s-3)^2 + 4}\,.
\end{aligned}$$

3. **Apply $\mathcal{L}^{-1}$:**

$$\begin{aligned}
y &= \mathcal{L}^{-1}\{Y\} \\
&= \mathcal{L}^{-1}\left\{ \left. \frac{-3}{(s-3)^2 + 4} \right|_{s \to s+3} \right\} \qquad \text{add 3 to all } s\text{'s} \\
&= e^{at}\,\mathcal{L}^{-1}\left\{ \frac{-3}{s^2 + 4} \right\}_{a=3} \qquad \text{by I-1TT} \\
&= e^{3t}\,\mathcal{L}^{-1}\left\{ \frac{-3}{s^2 + 4} \right\} \\
&= -3\,e^{3t}\,\mathcal{L}^{-1}\left\{ \frac{2}{s^2 + 2^2} \right\} \cdot \frac{1}{2} \qquad \text{pre-process} \\
&= -\frac{3}{2}\,e^{3t}\,\sin 2t\,.
\end{aligned}$$