Use Laplace transforms to solve the IVP

$$y'' - 2\,y' + y \;=\; e^t\,, \tag{1}$$

$$y(0) \;=\; 3\,, \qquad y'(0) \;=\; -4 \tag{2}$$

1. **Take $\mathcal{L}$ of ODE (1):**

$$\mathcal{L}\{y''\} - 2\,\mathcal{L}\{y'\} + \mathcal{L}\{y\} \;=\; \mathcal{L}\{e^t\}\,, \tag{3}$$

where

$$\begin{aligned} \mathcal{L}\{y\} &= Y\,, \\ \mathcal{L}\{y'\} &= s\,Y - y(0) \;=\; s\,Y - 3\,, \\ \mathcal{L}\{y''\} &= s^2\,Y - s\,y(0) - y'(0) \;=\; s^2\,Y - 3\,s + 4\,, \\ \mathcal{L}\{e^t\} &= \frac{1}{s-1}\,. \end{aligned}$$

Substitute these into Eq. (3) to obtain

$$s^2\,Y - 3\,s + 4 - 2\,(s\,Y - 3) + Y \;=\; \frac{1}{s-1}\,.$$

2. **Solve for Y:**

$$\begin{aligned} (s^2 - 2\,s + 1)\,Y &= \frac{1}{s-1} + 3\,s - 10\,, \\ (s-1)^2\,Y &= \frac{1}{s-1} + 3\,s - 10\,, \end{aligned}$$

so

$$Y \;=\; \frac{1}{(s-1)^3} \;+\; \frac{3s-10}{(s-1)^2}\,.$$

3. **Apply $\mathcal{L}^{-1}$:**

$$\begin{aligned} y &= \mathcal{L}^{-1}\{Y\} \\ &= \mathcal{L}^{-1}\left\{ \frac{1}{(s-1)^3} \;+\; \frac{3s-10}{(s-1)^2}\,\bigg|_{s\to s+1} \right\} \qquad \text{add 1 to all } s\text{'s} \\ &= e^{at}\,\mathcal{L}^{-1}\left\{ \frac{1}{s^3} \;+\; \frac{3(s+1)-10}{s^2} \right\}_{a=1} \qquad \text{by I-1TT} \\ &= e^t\,\mathcal{L}^{-1}\left\{ \frac{1}{s^3} \;+\; \frac{3}{s} \;-\; \frac{7}{s^2} \right\} \\ &= e^t\left[\mathcal{L}^{-1}\left\{ \frac{1}{s^3} \right\} \;+\; \mathcal{L}^{-1}\left\{ \frac{3}{s} \right\} \;-\; \mathcal{L}^{-1}\left\{ \frac{7}{s^2} \right\} \right] \\ &= e^t\left[\mathcal{L}^{-1}\left\{ \frac{2!}{s^3} \right\}\cdot\frac{1}{2!} \;+\; 3\,\mathcal{L}^{-1}\left\{ \frac{1}{s} \right\} \;-\; 7\,\mathcal{L}^{-1}\left\{ \frac{1}{s^2} \right\} \right] \\ &= e^t\left(\frac{1}{2}\,t^2 + 3 - 7\,t \right). \end{aligned}$$