Use Laplace transforms to solve the IVP

$$y'' - 2\,y' \;=\; 8\,e^{-2t}\,, \tag{1}$$

$$y(0) \;=\; 1\,, \qquad y'(0) \;=\; -4 \tag{2}$$

1. **Take $\mathcal{L}$ of ODE (1):**

$$\mathcal{L}\{y''\} - 2\,\mathcal{L}\{y'\} \;=\; 8\,\mathcal{L}\{e^{-2t}\}\,, \tag{3}$$

where

$$\begin{aligned}
\mathcal{L}\{y\} &= Y\,, \\
\mathcal{L}\{y'\} &= s\,Y - y(0) \;=\; s\,Y - 1\,, \\
\mathcal{L}\{y''\} &= s^2\,Y - s\,y(0) - y'(0) \;=\; s^2\,Y - s + 4\,, \\
\mathcal{L}\{e^{-2t}\} &= \frac{1}{s+2}\,.
\end{aligned}$$

Substitute these into Eq. (3) to obtain

$$s^2\,Y - s + 4 - 2\,(s\,Y - 1) \;=\; \frac{8}{s+2}\,.$$

2. **Solve for Y:**

$$\begin{aligned}
(s^2 - 2\,s)\,Y &= \frac{8}{s+2} + s - 6\,, \\
s\,(s-2)\,Y &= \frac{8}{s+2} + s - 6\,,
\end{aligned}$$

so

$$\begin{aligned}
Y &= \frac{8}{s\,(s-2)(s+2)} + \frac{s-6}{s(s-2)} \\
&= \frac{s^2 - 4s - 4}{s\,(s-2)(s+2)} \qquad \text{(now use p.f.d.)} \\
&= \frac{a}{s} + \frac{b}{s-2} + \frac{c}{s+2} \;=\; \cdots \text{ (you show) } \cdots \\
&= \frac{1}{s} - \frac{1}{s-2} + \frac{1}{s+2}\,.
\end{aligned}$$

3. **Apply $\mathcal{L}^{-1}$:**

$$\begin{aligned}
y &= \mathcal{L}^{-1}\{Y\} \\
&= \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} \\
&= 1 - e^{2t} + e^{-2t}\,.
\end{aligned}$$