Use Laplace transforms to solve the IVP

$$y' - y = \sin t, \tag{1}$$

$$y(0) = 0. \tag{2}$$

1. **Take $\mathcal{L}$ of ODE (1):**

$$\mathcal{L}\{y'\} - \mathcal{L}\{y\} = \mathcal{L}\{\sin t\}, \tag{3}$$

where

$$\begin{aligned} \mathcal{L}\{y\} &= Y, \\ \mathcal{L}\{y'\} &= sY - y(0) = sY - 0 = sY, \\ \mathcal{L}\{\sin t\} &= \frac{1}{s^2+1}. \end{aligned}$$

Substitute these into Eq. (3) to obtain

$$sY - Y = \frac{1}{s^2+1}.$$

2. **Solve for Y:**

$$(s-1)\,Y = \frac{1}{s^2+1},$$

so

$$\begin{aligned} Y &= \frac{1}{(s^2+1)(s-1)} \qquad \text{(use p.f.d.)} \\ &= \frac{as+b}{s^2+1} + \frac{c}{s-1} = \cdots \text{ (you show) } \cdots \\ &= -\frac{1}{2}\left(\frac{s}{s^2+1}\right) - \frac{1}{2}\left(\frac{1}{s^2+1}\right) + \frac{1}{2}\left(\frac{1}{s-1}\right). \end{aligned}$$

3. **Apply $\mathcal{L}^{-1}$:**

$$\begin{aligned} y &= \mathcal{L}^{-1}\{Y\} \\ &= -\frac{1}{2}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} - \frac{1}{2}\,\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + \frac{1}{2}\,\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} \\ &= -\frac{1}{2}\cos t - \frac{1}{2}\sin t + \frac{1}{2}\,e^t. \end{aligned}$$