Use Laplace transforms to solve the IVP

$$y' + 2\,y \;=\; t\,, \tag{1}$$

$$y(0) \;=\; -1\,. \tag{2}$$

1. **Take $\mathcal{L}$ of ODE (1):**

$$\mathcal{L}\{y'\} + 2\,\mathcal{L}\{y\} \;=\; \mathcal{L}\{t\}\,, \tag{3}$$

where

$$\begin{aligned} \mathcal{L}\{y\} &= Y\,, \\ \mathcal{L}\{y'\} &= sY - y(0) \;=\; sY - (-1) \;=\; sY + 1\,, \\ \mathcal{L}\{t\} &= \frac{1}{s^2}\,. \end{aligned}$$

Substitute these into Eq. (3) to obtain

$$sY + 1 + 2\,Y \;=\; \frac{1}{s^2}\,.$$

2. **Solve for Y:**

$$(s+2)\,Y \;=\; \frac{1}{s^2} - 1\,,$$

so

$$\begin{aligned} Y &= \frac{1}{s^2(s+2)} \;-\; \frac{1}{s+2} \qquad\qquad \text{(p.f.d. the first term)} \\ &= -\frac{1}{4}\left(\frac{1}{s}\right) \;+\; \frac{1}{2}\left(\frac{1}{s^2}\right) \;+\; \frac{1}{4}\left(\frac{1}{s+2}\right) \;-\; \left(\frac{1}{s+2}\right) \\ &= -\frac{1}{4}\left(\frac{1}{s}\right) \;+\; \frac{1}{2}\left(\frac{1}{s^2}\right) \;-\; \frac{3}{4}\left(\frac{1}{s+2}\right). \end{aligned}$$

3. **Apply $\mathcal{L}^{-1}$:**

$$\begin{aligned} y &= \mathcal{L}^{-1}\{Y\} \\ &= -\frac{1}{4}\,\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} \;+\; \frac{1}{2}\,\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} \;-\; \frac{3}{4}\,\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} \\ &= -\frac{1}{4} \;+\; \frac{1}{2}\,t \;-\; \frac{3}{4}\,e^{-2t}\,. \end{aligned}$$