A circuit comprises a 2 henry inductor, a 20 Ω resistor, and a 1/338 farad capacitor. The initial charge stored on the capacitor is -4 Coulombs and the initial current is $+56$ amps. Obtain the solution for charge $q(t)$.

Given:

$$L = 2 \text{ h}, \qquad C = 1/338 \text{ f}, \qquad R = 20\,\Omega,$$
$$q(0) = -4, \qquad q'(0) = i(0) = +56.$$

There is no voltage source, so the ODE (governing equation) is then

$$L\,q'' + R\,q' + \frac{1}{C}\,q \;=\; 0 \qquad \Longrightarrow \qquad 2\,q'' + 20\,q' + 338\,q \;=\; 0\,. \tag{1}$$

The characteristic equation

$$m^2 + 10m + 169 \;=\; 0$$

has roots

$$m_{1,2} \;=\; -5 \pm 12i\,,$$

so the general solution is

$$q(t) \;=\; e^{-5t}(c_1\,\cos 12t + c_2\,\sin 12t)\,. \tag{2}$$

Then

$$q'(t) \;=\; -5\,e^{-5t}(c_1\,\cos 12t + c_2\,\sin 12t) + e^{-5t}(-12c_1\,\sin 12t + 12c_2\,\cos 12t)\,. \tag{3}$$

Initial Conditions:

From equations (2) and (3) we obtain

$$q(0) \;=\; e^0(c_1\,\cos 0 + c_2\,\sin 0)\,,$$
$$-4 \;=\; c_1\,, \tag{4}$$
$$q'(0) \;=\; -5\,e^0(c_1\,\cos 0 + c_2\,\sin 0) + e^0(-12c_1\,\sin 0 + 12c_2\,\cos 0)\,,$$
$$+56 \;=\; -5c_1 + 12c_2\,. \tag{5}$$

We solve equations (4) and (5) for c_1 and c_2 to obtain $c_1 = -4$ and $c_2 = 3$. So the solution is

$$q(t) \;=\; e^{-5t}(-4\cos 12t + 3\sin 12t)\,. \tag{6}$$

NOTE: The period is

$$T \;=\; \frac{2\pi}{12} \;=\; \frac{\pi}{6} \;\text{ sec/cycle}\,.$$

NOTES:

1. Recall that the current is
$$i(t) \;\equiv\; q'(t) \;=\; \cdots$$
2. This is exactly Example 4 handed out in Section 5.1.2.