A 2 kg mass is attached to a spring with spring constant of 8 N/m. The spring is initially 3 m above equilibrium and given an upward velocity of 8 m/s. Obtain the equation of motion.

Given:

$$m = 2 \text{ kg}, \qquad k = 8 \text{ N/m}, \qquad \beta = 0,$$
$$x(0) = -3 \text{ m}, \qquad x'(0) = -8 \text{ m/s}.$$

The ODE (governing equation) is then

$$m\,x'' + \beta\,x' + k\,x \;=\; 0 \qquad \Longrightarrow \qquad 2\,x'' + 8\,x \;=\; 0\,. \tag{1}$$

The characteristic equation

$$m^2 + 4 \;=\; 0$$

has roots

$$m_{1,2} \;=\; \pm 2i\,.$$

Since the roots are complex, the general solution of this homogeneous ODE is *simple harmonic*:

$$x(t) \;=\; c_1\,\cos 2t + c_2\,\sin 2t\,, \tag{2}$$

So

$$x'(t) \;=\; -2c_1\,\sin 2t + 2c_2\,\cos 2t\,. \tag{3}$$

Initial Conditions:

From equations (2) and (3) we obtain

$$\begin{aligned} x(0) &= c_1\,\cos 0 + c_2\,\sin 0\,, \\ -3 &= c_1\,, \end{aligned} \tag{4}$$

$$\begin{aligned} x'(0) &= -2c_1\,\sin 0 + 2c_2\,\cos 0\,, \\ -8 &= 2c_2\,. \end{aligned} \tag{5}$$

We solve equations (4) and (5) for c_1 and c_2 to obtain $c_1 = -3$ and $c_2 = -4$. So the solution, now called the *equation of motion*, is obtained by Eq. (2):

$$x(t) \;=\; -3\cos 2t - 4\sin 2t\,. \tag{6}$$

NOTE: The *period* is the time it takes to complete one cycle (vibration, oscillation):

$$T \;=\; \frac{2\pi}{2} \;=\; \pi \text{ seconds/cycle}\,, \qquad \text{(takes } \pi \text{ seconds to complete each cycle)}$$

The *frequency* is number of cycles completed every second:

$$f \;=\; \frac{1}{T} \;=\; \frac{1}{\pi} \text{ cycles/sec}\,, \qquad \text{(completes } 1/\pi \text{ cycles per second)}$$