The method of **Variation of Parameters** is used to construct a **particular solution** y_p of a 2nd–order, linear, **nonhomogeneous** ODE of the form:

$$1 y'' + a_1\, y' + a_0\, y \;=\; f(x)\,, \tag{N}$$

where the coefficients a_1 and a_0 may be functions of x. (This method may be generalized to solve higher order ODEs.)

IMPORTANT: note that the first term **must** be a lone y''.

STEP 1:
Obtain the two linearly independent solutions y_1 and y_2 of the *associated homogeneous problem*

$$y'' + a_1\, y' + a_0\, y \;=\; 0\,. \tag{H}$$

We did this in Section 4.3 to obtain the *complementary solution* of (N):

$$y_c \;=\; c_1\, y_1 + c_2\, y_2\,. \tag{C}$$

STEP 2: Use **Variation of Parameters** to construct a *particular solution* y_p of problem (N):

(a) Set:

$$W \;\equiv\; \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \;=\; y_1\, y_2' - y_1'\, y_2\,,$$

$$W_1 \;\equiv\; \begin{vmatrix} 0 & y_2 \\ f & y_2' \end{vmatrix} \;=\; -y_2\, f\,, \qquad W_2 \;\equiv\; \begin{vmatrix} y_1 & 0 \\ y_1' & f \end{vmatrix} \;=\; +y_1\, f\,.$$

(b) Set

$$u_1 \;\equiv\; \int \frac{W_1}{W}\, dx\,, \qquad u_2 \;\equiv\; \int \frac{W_2}{W}\, dx\,.$$

(c) Then a *particular solution* of (N) is

$$y_p \;=\; u_1\, y_1 + u_2\, y_2\,. \tag{P}$$

Simplify y_p if possible.

STEP 3: Then the *general solution* of the nonhomogeneous problem (N) is

$$y \;=\; y_c + y_p\,. \tag{G}$$

Simplify y if possible.

STEP 4: Apply any initial or boundary conditions to the general solution (G), **NOT** to the complementary solution (C).