The method of **Undetermined Coefficients** is used to construct a **particular solution** of a linear, **non**homogeneous ODE with constant coefficients:

$$a_n\, y^{(n)} + a_{n-1}\, y^{(n-1)} + \cdots + a_2\, y'' + a_1\, y' + a_0\, y \;=\; g(x) \qquad \textbf{(N)}$$

when $g(x)$ is any linear combination of functions of the form

$$k, \quad x^m, \quad e^{\alpha x}, \quad x^m\, e^{\alpha x}, \quad e^{\alpha x}\cos\beta x, \quad e^{\alpha x}\sin\beta x, \quad x^m\, e^{\alpha x}\cos\beta x, \quad x^m\, e^{\alpha x}\sin\beta x\,.$$

The function $g(x)$ is often called the *input function*, the *forcing function*, or the *driving force*.

STEP 1:
Obtain the general solution of the *associated homogeneous problem*

$$a_n\, y^{(n)} + a_{n-1}\, y^{(n-1)} + \cdots + a_2\, y'' + a_1\, y' + a_0\, y \;=\; 0\,. \qquad \textbf{(H)}$$

We did this in Section 4.3 to obtain

$$y_c \;=\; c_1\, y_1 + c_2\, y_2 + \cdots + c_n\, y_n\,. \qquad \textbf{(C)}$$

This general solution of (H) is called the *complementary solution* of (N), so we now denote it by y_c.

Notes:

1. y_c is the general solution of (H), not of (N).
2. The functions $y_1, \ldots, y_n$ are linearly independent solutions of (H), not of (N).

STEP 2:
Use **Undetermined Coefficients** to construct a *particular solution* y_p of the nonhomogeneous problem (N). This is the method we'll learn in Section 4.4.

STEP 3:
Then the general solution of the nonhomogeneous problem (N) is

$$y \;=\; y_c + y_p\,. \qquad \textbf{(G)}$$

It is an n parameter family of solutions of (N).

STEP 4:
If there are any initial conditions, apply them to the general solution (G), **NOT** to solution (C)!