

**We will use the result from this problem
in subsequent examples.**

Solve the homogeneous ODE

$$y'' - y' - 6y = 0 \quad (\text{H})$$

Characteristic Equation:

$$\begin{aligned} m^2 - m - 6 &= 0 \\ \implies (m - 3)(m + 2) &= 0 \\ \implies m_1 = +3, \quad m_2 = -2 \end{aligned}$$

So 2 linearly independent solutions of (H) are

$$\begin{aligned} m_1 = +3 &\rightarrow y_1 = e^{3x} \\ m_2 = -2 &\rightarrow y_2 = e^{-2x} \end{aligned}$$

So the general solution of (H) is

$$\begin{aligned} y &= c_1 y_1 + c_2 y_2 \\ &= c_1 e^{3x} + c_2 e^{-2x} \end{aligned}$$