Factoring by Synthetic Division **Dr. K.G. TeBeest**

To factor a polynomial. Review factoring in your algebra or calculus text if necessary.

1. Factor $P(m) = m^3 - 10\,m^2 + 31\,m - 30.$

   (a) First determine a root of P by trial and error. That is, determine a value of m for which $P = 0$. Try $m = 0$, $m = 1$, $m = -1$, $m = 2$, $m = -2$, $m = 3$, *etc.*

   (b) Since $m = 2$ is a root of P, then $(m - 2)$ is a factor of P.

   (c) Since P is 3rd degree, the remaining factor must be 2nd degree. To determine the remaining factor, use synthetic division. In row 1, place the root in the first place, and place the coefficients of P in the remaining places.

   (d) Drag the first coefficient straight down to row 3. Multiply it by the root, and place the result in row 2 below the 2nd coefficient.

   (e) Add these numbers, and place the result in row 3.

   (f) Multiply that result by the root, and place the result in row 2 below the 3rd coefficient. *etc.*

$$\begin{array}{c|cccc} \boxed{2} & 1 & -10 & 31 & -30 \\ & + & 2 & -16 & 30 \\ \hline & 1 & -8 & 15 & \boxed{0} \end{array}$$

   NOTE: In row 3, the last value *must* be a zero. If it isn't, then there's an error somewhere.

   Furthermore, the other numbers in row 3 are the coefficients of the remaining factor, which we already said must be 2nd degree. So

$$P(m) = (m - 2)(1\,m^2 - 8\,m + 15) \;=\; (m - 2)(m - 3)(m - 5)\,.$$

2. Factor $P(m) = 2\,m^3 - 7\,m^2 + 9.$

   By trial and error, we find that $m = -1$ is a root of P, so $(m + 1)$ is a factor of P. Since P is 3rd degree, the remaining factor must be 2nd degree.

$$\begin{array}{c|cccc} \boxed{-1} & 2 & -7 & 0 & 9 \\ & + & -2 & 9 & -9 \\ \hline & 2 & -9 & 9 & \boxed{0} \end{array}$$

   The last value *is* zero. The remaining numbers are the coefficients of the remaining factor, which we already said must be 2nd degree. So

$$P(m) = (m + 1)(2\,m^2 - 9\,m + 9) \;=\; (m + 1)(2m - 3)(m - 3)\,.$$

3. Factor $P(m) = m^3 + 2\,m^2 - 5\,m - 6$.

   By trial and error, we find that $m = 2$ is a root of P, so $(m-2)$ is a factor of P. Since P is 3rd degree, the remaining factor must be 2nd degree.

$$\begin{array}{c|cccc} \boxed{2} & 1 & 2 & -5 & -6 \\ & + & 2 & 8 & 6 \\ \hline & 1 & 4 & 3 & \boxed{0} \end{array}$$

   The last value *is* zero. The remaining numbers are the coefficients of the remaining factor, which we already said must be 2nd degree. So

$$P(m) = (m-2)(1\,m^2 + 4\,m + 3) \;=\; (m-2)(m+3)(m+1)\,.$$

4. Factor $P(m) = m^3 - 7\,m - 6$. Note that there is no m^2 term, so the poynomial must be viewed as $P(m) = m^3 + 0\,m^2 - 7\,m - 6$.

   By trial and error, we find that $m = 3$ is a root of P, so $(m-3)$ is a factor of P. Since P is 3rd degree, the remaining factor must be 2nd degree.

$$\begin{array}{c|cccc} \boxed{3} & 1 & 0 & -7 & -6 \\ & + & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & \boxed{0} \end{array}$$

   The last value *is* zero. The remaining numbers are the coefficients of the remaining factor, which we already said must be 2nd degree. So

$$P(m) = (m-3)(1\,m^2 + 3\,m + 2) \;=\; (m-3)(m+1)(m+2)\,.$$

5. Factor $P(m) = m^3 + 9\,m^2 + 3\,m - 22$.

   By trial and error, we find that $m = -2$ is a root of P, so $(m+2)$ is a factor of P. Since P is 3rd degree, the remaining factor must be 2nd degree.

$$\begin{array}{c|cccc} \boxed{-2} & 1 & 9 & 3 & -22 \\ & + & -2 & -14 & 22 \\ \hline & 1 & 7 & -11 & \boxed{0} \end{array}$$

   The last value *is* zero. The remaining numbers are the coefficients of the remaining factor, which we already said must be 2nd degree. So

$$P(m) = (m+2)(1\,m^2 + 7\,m - 11)\,.$$

   In this case, the second factor $(m^2 + 7\,m - 11)$ cannot be factored further.