Solve the homogeneous ODE

$$y''' + 2y'' - 5y' - 6y \;=\; 0. \tag{H}$$

This ODE is 3rd order, so we must determine 3 linearly independent solutions.

**Characteristic Equation:**

$$m^3 + 2m^2 - 5m - 6 \;=\; 0$$

$$\Longrightarrow \;\; (m+1)(m-2)(m+3) \;=\; 0$$

$$\Longrightarrow \;\; m_1 = -1, \quad m_2 = 2, \quad m_3 = -3.$$

These roots are real and distinct.

So 3 linearly independent solutions of (H) are

$$m_1 = -1 \;\; \rightarrow \;\; y_1 = e^{-x},$$

$$m_2 = +2 \;\; \rightarrow \;\; y_2 = e^{2x},$$

$$m_3 = -3 \;\; \rightarrow \;\; y_3 = e^{-3x}.$$

So the general solution of (H) is

$$\begin{aligned} y \;&=\; c_1\,y_1 + c_2\,y_2 + c_3\,y_3 \\[1mm] &=\; c_1\,e^{-x} + c_2\,e^{2x} + c_3\,e^{-3x}. \end{aligned}$$

This is a 3–parameter family of solutions.