Solve the homogeneous ODE

$$4y'' + 20y' + 25y \;=\; 0\,. \qquad \textbf{(H)}$$

Characteristic Equation:

$$\begin{aligned}
& 4m^2 + 20m + 25 \;=\; 0 \\
\Longrightarrow\quad & (2m+5)^2 \;=\; 0 \\
\Longrightarrow\quad & (2m+5)(2m+5) \;=\; 0 \\
\Longrightarrow\quad & m_1 = -5/2, \quad m_2 = -5/2\,.
\end{aligned}$$

These roots are real and equal. I say the root *appears once* in m_1 and *repeats once* in m_2.

So 2 linearly independent solutions of (H) are

$$\begin{aligned}
m_1 = -5/2 \;\;&\rightarrow\;\; y_1 = e^{-5x/2}\,, \\
m_2 = -5/2 \;\;&\rightarrow\;\; y_2 = xe^{-5x/2}\,.
\end{aligned}$$

So the general solution of (H) is

$$\begin{aligned}
y \;&=\; c_1\,y_1 + c_2\,y_2 \\
&=\; c_1\,e^{-5x/2} + c_2\,x\,e^{-5x/2} \qquad \text{or} \\
&=\; e^{-5x/2}(c_1 + c_2\,x)\,.
\end{aligned}$$

This is a 2–parameter family of solutions.