Solve the homogeneous ODE

$$y'' - 3y' - 10y \;=\; 0\,. \qquad \textbf{(H)}$$

Characteristic Equation:

$$\begin{aligned} & m^2 - 3m - 10 \;=\; 0 \\ \Longrightarrow\;\; & (m-5)(m+2) \;=\; 0 \\ \Longrightarrow\;\; & m_1 = +5, \quad m_2 = -2\,. \end{aligned}$$

These roots are real and distinct.

So 2 linearly independent solutions of (H) are

$$\begin{aligned} m_1 = +5 \;\;&\rightarrow\;\; y_1 = e^{5x} \\ m_2 = -2 \;\;&\rightarrow\;\; y_2 = e^{-2x}\,. \end{aligned}$$

So the general solution of (H) is

$$\begin{aligned} y \;\;&=\;\; c_1\, y_1 + c_2\, y_2 \\ &=\;\; c_1\, e^{5x} + c_2\, e^{-2x}\,. \end{aligned}$$

This is a 2–parameter family of solutions.