Partial fraction decomposition has many uses in mathematics, so it is important to become expert at decomposing rational functions. To decompose a rational function

$$f(x) \;=\; \frac{N(x)}{D(x)}\,,$$

where both $N(x)$ and $D(x)$ are polynomials with $\deg(N) < \deg(D)$.

1. Fully factor the denominator:

   $$D(x) \;=\; r_1(x)\, r_2(x) \cdots r_n(x)\,,$$

   where the r factors are irreducible.

2. Each r factor in the denominator generates one term in the decomposition of f.

3. Above each r factor, place a general polynomial that is one degree smaller than the degree of r. Namely,

   (a) above a linear r factor, place a constant term a,

   (b) above a quadratic r factor, place a linear term $(ax + b)$.

Use partial fraction decomposition to decompose the following rational functions.

FUNCTION	**FORM**	**DECOMPOSITION**
$\dfrac{4-3x}{2x^2+x}$	$\dfrac{a}{x}+\dfrac{b}{2x+1}$	$\dfrac{4}{x}-\dfrac{11}{2x+1}$
$\dfrac{5-2x}{x^2-3x+2}$	$\dfrac{a}{x-2}+\dfrac{b}{x-1}$	$\dfrac{1}{x-2}-\dfrac{3}{x-1}$
$\dfrac{36-2x}{x^2-9}$	$\dfrac{a}{x-3}+\dfrac{b}{x+3}$	$\dfrac{5}{x-3}-\dfrac{7}{x+3}$
$\dfrac{-2\,x^2+7\,x+4}{x^3+x^2}$	$\dfrac{a}{x}+\dfrac{b}{x^2}+\dfrac{c}{x+1}$	$\dfrac{3}{x}+\dfrac{4}{x^2}-\dfrac{5}{x+1}$
$\dfrac{2\,x+4}{x^3-4\,x^2+4\,x}$	$\dfrac{a}{x}+\dfrac{b}{x-2}+\dfrac{c}{(x-2)^2}$	$\dfrac{1}{x}-\dfrac{1}{x-2}+\dfrac{4}{(x-2)^2}$
$\dfrac{9\,x^2+4\,x+54}{x^3+9\,x}$	$\dfrac{a}{x}+\dfrac{bx+c}{x^2+9}$	$\dfrac{6}{x}+\dfrac{3x+4}{x^2+9}$