An **integrating factor** is used to solve a first order, linear ODE of the form:

$$\frac{dy}{dx} + P(x)y \ = \ f(x)\,. \tag{1}$$

1. Set
$$r \ = \ \int P(x)\,dx\,. \qquad \text{no } +c$$

2. The *integrating factor* is then
$$\mu \ = \ e^r \qquad\qquad \text{(simplify)} \tag{2}$$

3. Multiply Eq. (1) by function μ to obtain
$$\mu\frac{dy}{dx} + \mu P(x)\,y \ = \ \mu f(x)\,. \tag{3}$$

   Litmus Test: The LHS of (3) should be $\dfrac{d}{dx}(\mu y)$. Confirm this.

   Then Eq. (3) may be written as
$$\frac{d}{dx}(\mu y) \ = \ \mu f(x)\,. \tag{4}$$

4. Then integrate Eq. (4) to obtain
$$\mu y \ = \ \int \mu f(x)\;dx \ + \ c\,. \tag{5}$$

   Divide by μ and substitute μ to obtain the solution in explicit form:
$$y \ = \ \text{ftn}(x)\,.$$

5. If there is an initial condition $y(x_0) = y_0$, apply it to the solution to determine the constant c.

NOTE: If the ODE has the form
$$\frac{dx}{dy} + P(y)x \ = \ f(y)\,,$$
then merely:

A. Interchange $x \longleftrightarrow y$ to obtain
$$\frac{dy}{dx} + P(x)y \ = \ f(x)\,.$$

B. Apply Steps 1–4 to solve this ODE for
$$y \ = \ \text{ftn}(x)\,.$$

C. Interchange $x \longleftrightarrow y$ to obtain the solution
$$x \ = \ \text{ftn}(y)\,.$$