

It is crucial that you master Section 4.3, so these are additional practice problems. Do these immediately.

Obtain the general solution of each ODE (as in Section 4.3, of course).

## Differential Equation

## General Solution

1.  $y'' + y' - 6y = 0$

$y = c_1 e^{2x} + c_2 e^{-3x}$

2.  $y''' + y'' - 6y' = 0$

$y = c_1 + c_2 e^{2x} + c_3 e^{-3x}$

3.  $y^{(5)} + y^{(4)} - 6y^{(3)} = 0$

$y = c_1 + c_2 x + c_3 x^2 + c_4 e^{2x} + c_5 e^{-3x}$

4.  $y'' - 9y = 0$

$y = c_1 e^{3x} + c_2 e^{-3x}$

5.  $y'' + 9y = 0$

$y = c_1 \cos 3x + c_2 \sin 3x$

6.  $y''' - 9y' = 0$

$y = c_1 + c_2 e^{3x} + c_3 e^{-3x}$

7.  $y^{(4)} + 9y'' = 0$

$y = c_1 + c_2 x + c_3 \cos 3x + c_4 \sin 3x$

8.  $y'' + 2y' + 2y = 0$

$y = c_1 e^{-x} \cos x + c_2 e^{-x} \sin x$

9.  $y'' - 6y' + 9y = 0$

$y = c_1 e^{3x} + c_2 x e^{3x}$

10.  $y^{(6)} - 6y^{(5)} + 9y^{(4)} = 0$

$y = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + c_5 e^{3x} + c_6 x e^{3x}$

11.  $y''' + 2y'' - 5y' - 6y = 0$

$y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-3x}$

12.  $y^{(4)} + 18y'' + 81y = 0$

$y = c_1 \cos 3x + c_2 \sin 3x + c_3 x \cos 3x + c_4 x \sin 3x$

13.  $y'' + 4y' + 13y = 0$

$y = c_1 e^{-2x} \cos 3x + c_2 e^{-2x} \sin 3x$

14.  $y''' + 4y'' + 13y' = 0$

$y = c_1 + c_2 e^{-2x} \cos 3x + c_3 e^{-2x} \sin 3x$

15.  $y''' + 5y'' + 17y' + 13y = 0$

$y = c_1 e^{-x} + c_2 e^{-2x} \cos 3x + c_3 e^{-2x} \sin 3x$

16. Suppose a linear, homogeneous ODE has a characteristic equation whose roots are

$$m = 0, 0, 0, 0, 2, 2, \pm 3i, \pm 3i.$$

Write the general solution.

Answer:

$$y = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + c_5 e^{2x} + c_6 x e^{2x} \\ + c_7 \cos 3x + c_8 \sin 3x + c_9 x \cos 3x + c_{10} x \sin 3x$$