A fin is a long and thin protrusion extending from a surface used to transfer heat.

Consider a cylindrical fin of radius $r$ and length 1. The fin’s base is attached to a hot surface whose temperature is fixed at $T_b$, and the surrounding air is maintained at a cooler temperature $T_o$. The temperature $T = T(x)$ in the fin decreases with increased distance $x$ from the base. The fin’s length is so much greater than its thickness that we may assume that its temperature depends only on $x$ (the distance from the base).

The equation governing the temperature variation within the fin is

$$\frac{d^2T}{dx^2} = \omega^2(T - T_o), \quad 0 < x < 1, \quad (1)$$

$$T(0) = T_b, \quad T(1) = T_o, \quad (2)$$

where

$$k - \text{thermal conductivity, units W/mK}$$

$$h - \text{convection coefficient, units W/m}^2\text{K}$$

$$\omega^2 = \frac{2h}{r k}$$

Note: $k$, $h$, $r$, and $\omega$ are positive constants.

1. Use **undeterminate coefficients** to obtain the general solution $T(x)$ of ODE (1). The general solution will also involve constants $\omega$ and $T_o$.

2. Apply the boundary conditions (2) to obtain the temperature $T$ explicitly in terms of $x$. The solution will also involve constants $\omega$, $T_b$, and $T_o$. Then do the necessary algebra to show that the solution may be written as

$$T(x) = T_o + \frac{T_b - T_o}{e^{\omega} - e^{-\omega}} \cdot \left[ e^{\omega(1-x)} - e^{\omega(x-1)} \right].$$

This gives the temperature in the fin at location $x$ measured from the fin’s base.

You will study fins extensively if you take the first course in Heat Transfer.