A **Bernoulli equation** is a first order ODE of the form:

$$1y' + P(x)\,y \;=\; f(x)\,\underline{y^n}\,, \qquad n \neq 0, 1 \tag{1}$$

1. Divide Eq. (1) by y^n (multiply by y^{-n}) to obtain

$$\boxed{y^{-n}\,y'} + P(x)\,\boxed{y^{1-n}} \;=\; f(x)\,. \tag{2}$$

2. Set

$$w \;=\; \boxed{y^{1-n}}\,, \tag{3}$$

$$\text{so}\quad w' \;=\; (1-n)\,y^{-n}\,y' \quad\Longrightarrow\quad \boxed{y^{-n}\,y'} \;=\; \frac{1}{1-n}\,w'\,. \tag{4}$$

3. Substitute Eqs. (3) and (4) into Eq. (2) to obtain

$$\frac{1}{1-n}\,w' + P(x)\,w \;=\; f(x)\,.$$

   Multiply this by $(1-n)$ to obtain

$$1w' + (1-n)\,P(x)\,w \;=\; (1-n)\,f(x)\,. \tag{5}$$

   This ODE is linear, and so can be solved for function w using *integrating factors* to obtain

$$w \;=\; \text{ftn}(x) + c\,. \tag{6}$$

4. Then subst. w from Eq. (3) into this result, and simplify it (write it in explicit form, if possible).
5. If there is an initial condition $y(x_0) = y_0$, apply it to the solution to determine the constant c.

DON'T MEMORIZE THE FORMULAS. LEARN THE PROCESS!

NOTE: If $x = x(y)$ and the ODE has the form

$$\frac{dx}{dy} + P(y)\,x \;=\; f(y)\,x^n\,, \tag{7}$$

then merely:

A. Interchange $x \longleftrightarrow y$ to obtain

$$\frac{dy}{dx} + P(x)\,y \;=\; f(x)\,y^n\,. \tag{8}$$

B. Apply Steps 1–4 to solve ODE (8).

C. Interchange $x \longleftrightarrow y$ to obtain the solution of Eq. (7).