Ex. Use polar coords to find the volume of the solid bound:

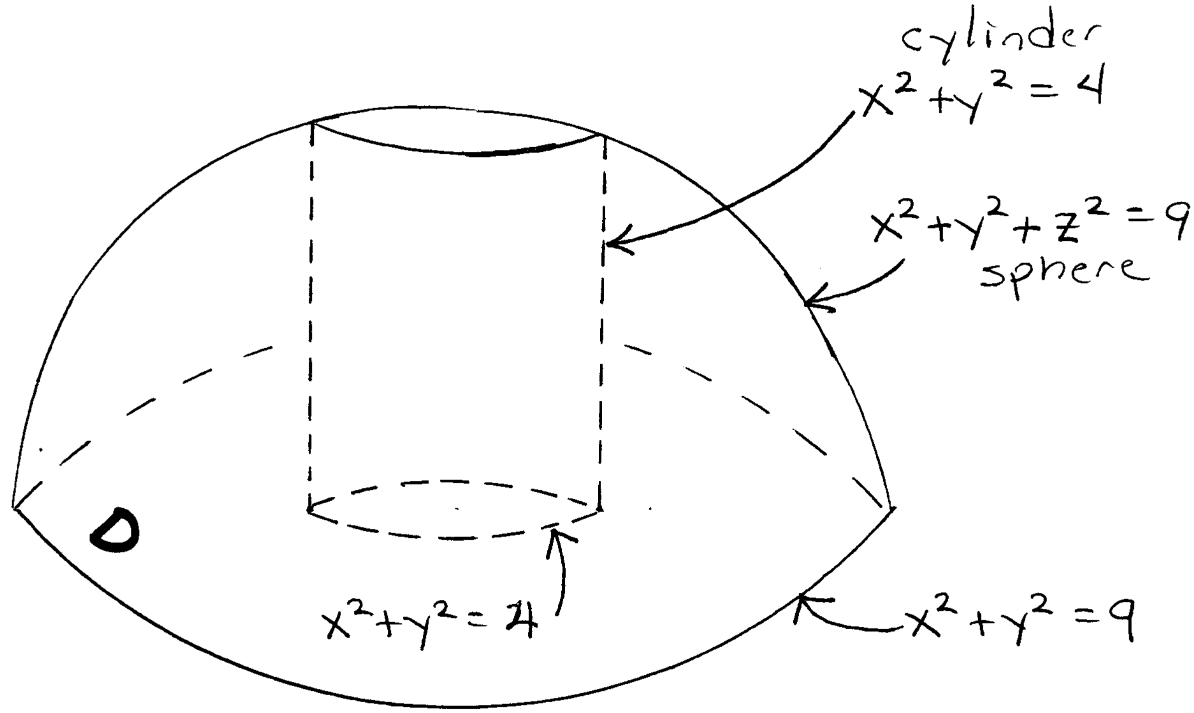
inside the sphere $x^2+y^2+z^2=9$, &

outside the cylinder $x^2+y^2=4$.

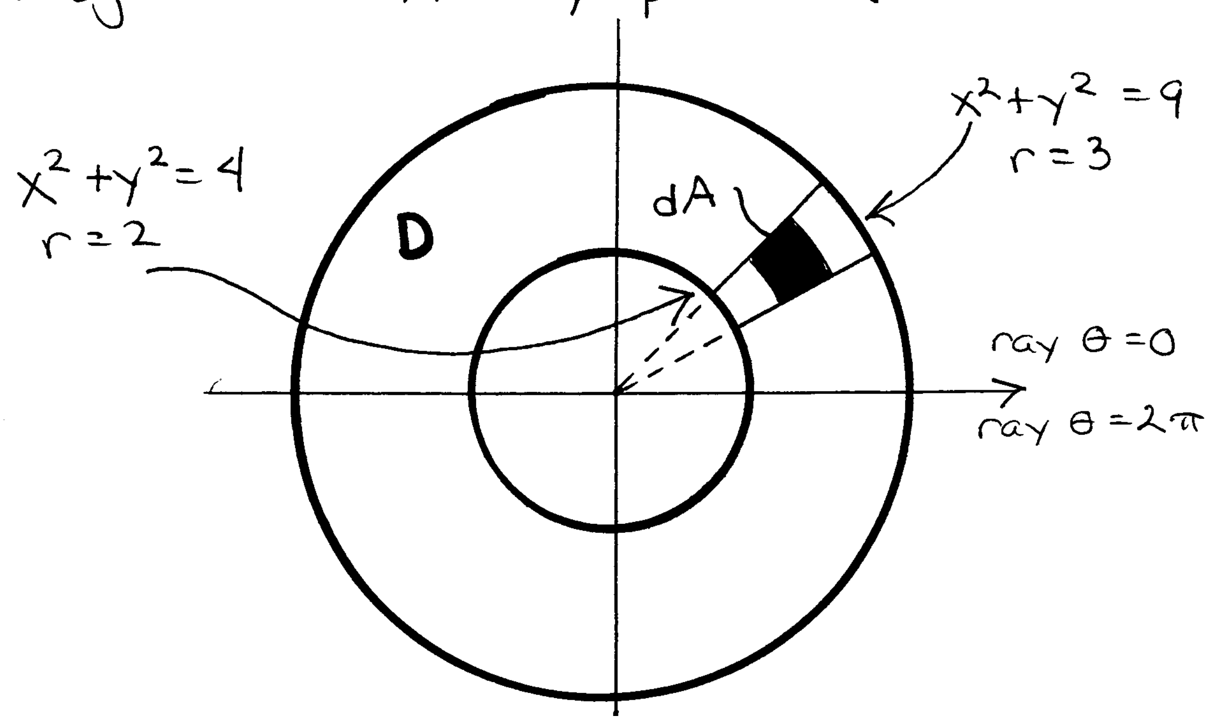
Sphere has radius 3

cylinder has radius 2

Upper Half:



Projected onto xy-plane (view from above):



Outer "rectangle" rotates from (thickness = $r\,d\theta$)
$\theta = 0$ (LL) to $\theta = 2\pi$ (UL)

Inner "rectangle" slides from (thickness = dr)
$r = 2$ (LL) to $r = 3$ (UL).

Surface: $x^2 + y^2 + z^2 = 9$

$$\Rightarrow z^2 = 9 - x^2 - y^2$$

$$\Rightarrow z = (9 - x^2 - y^2)^{1/2} = f(x,y)$$

So

$$V = 2 \iint_D f(x,y)\, dA$$

twice the ~~[illegible]~~ upper half

$$= 2 \iint_D (9 - x^2 - y^2)^{1/2}\, dA$$

$$= 2 \int_0^{2\pi} \int_2^3 (9 - r^2)^{1/2} \cdot \overbrace{(r\,dr\,d\theta)}^{dA}$$

$$u = 9 - r^2$$

$$\frac{du}{dr} = -2r \Rightarrow r\,dr = -\tfrac{1}{2}\,du$$

LL: $r = 2 \Rightarrow u = 9 - 2^2 = 5$
UL: $r = 3 \Rightarrow u = 9 - 3^2 = 0$

Inner integral:

$$\int_2^3 (9 - r^2)^{1/2}\, r\,dr \overset{\text{sub}}{=} \int_5^0 u^{\frac{1}{2}} \left(-\tfrac{1}{2}\,du\right)$$

$$= +\frac{1}{2}\int_0^5 u^{1/2}\,du$$

$$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_{u=0}^{5}$$

$$= \frac{1}{3}\left(5^{3/2} - 0^{3/2}\right) = \frac{5}{3}\sqrt{5}$$

Outer integral:

$$V = 2\int_0^{2\pi} \frac{5}{3}\sqrt{5}\,d\theta$$

$$= \frac{10}{3}\sqrt{5}\int_0^{2\pi} d\theta$$

$$= \frac{10}{3}\sqrt{5}\cdot\theta\Big|_{\theta=0}^{2\pi}$$

$$= \frac{10}{3}\sqrt{5}\left(2\pi - 0\right)$$

$$= \frac{20}{3}\sqrt{5}\,\pi \text{ units}^3$$