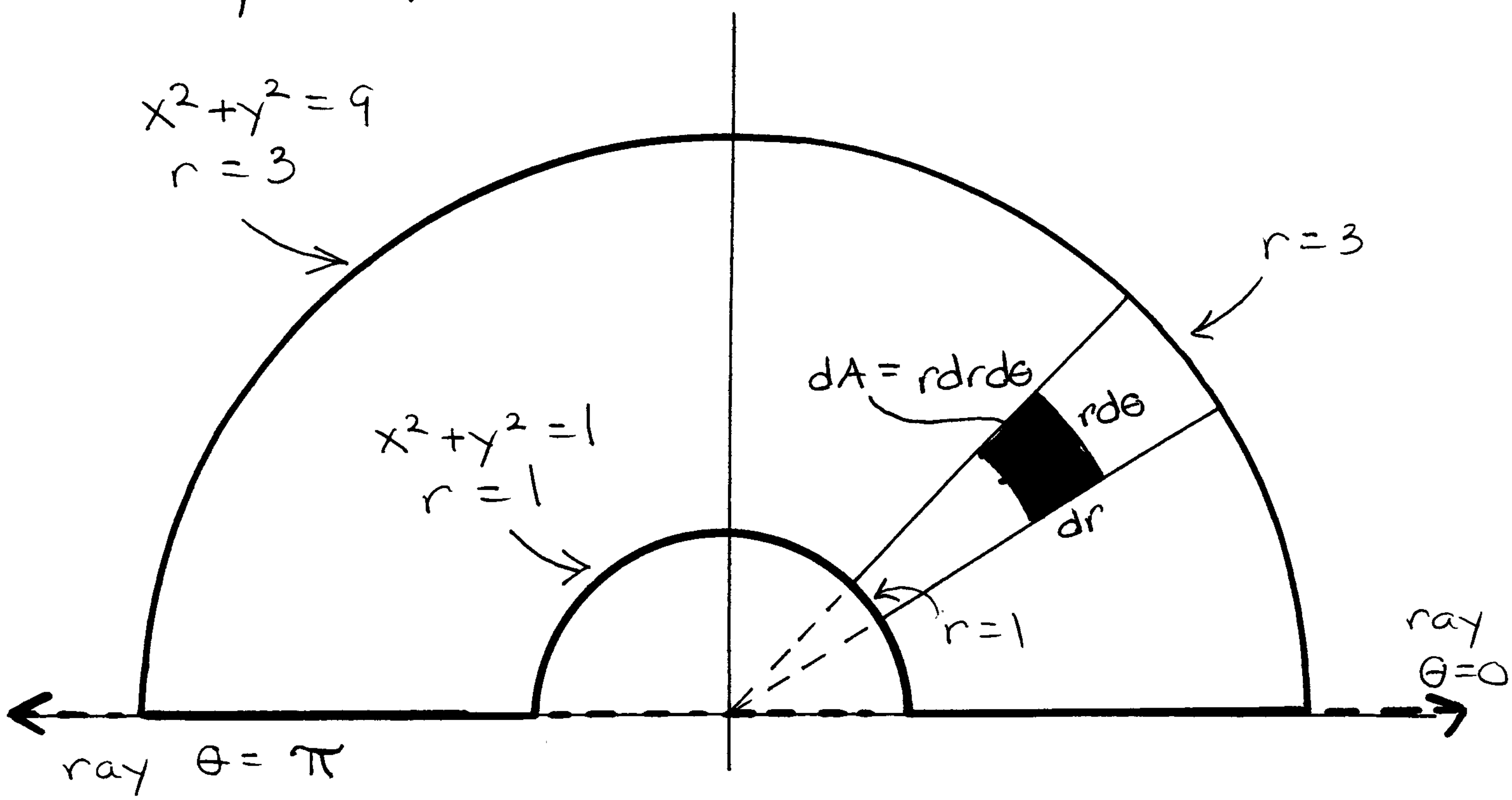
Ex. Evaluate $\iint_R \sqrt{x^2+y^2}\, dA$ if R is the region

$$R = \{(x,y) \mid 1 \le x^2+y^2 \le 9,\ y \ge 0\}.$$

So R is the region in the upper half-plane between circles $x^2+y^2=1$ & $x^2+y^2=9$.



Outer "rectangle" rotates from (thickness $= r\,d\theta$)
$\theta = 0$ (LL) to $\theta = \pi$ (UL)

Inner "rectangle" slides from (thickness $= dr$)
$r = 1$ (LL) to $r = 3$ (UL)

Furthermore,

$$f(x,y) = \sqrt{x^2+y^2} = \sqrt{r^2} = r$$

So
$$\iint_R \sqrt{x^2+y^2}\, dA$$

$$= \int_0^{\pi} \int_1^3 r \; \overbrace{(r\,dr\,d\theta)}^{dA}$$

$$= \int_0^{\pi} \int_1^3 r^2\,dr \; d\theta$$

$$= \int_0^{\pi} \frac{1}{3} r^3 \Big|_{r=1}^{3} \; d\theta$$

$$= \int_0^{\pi} \frac{1}{3}\left(3^3 - 1^3\right) d\theta$$

$$= \frac{26}{3} \int_0^{\pi} d\theta$$

$$= \frac{26}{3}\, \theta \Big|_{\theta=0}^{\pi}$$

$$= \frac{26}{3}\,(\pi - 0)$$

$$= \frac{26}{3}\,\pi$$