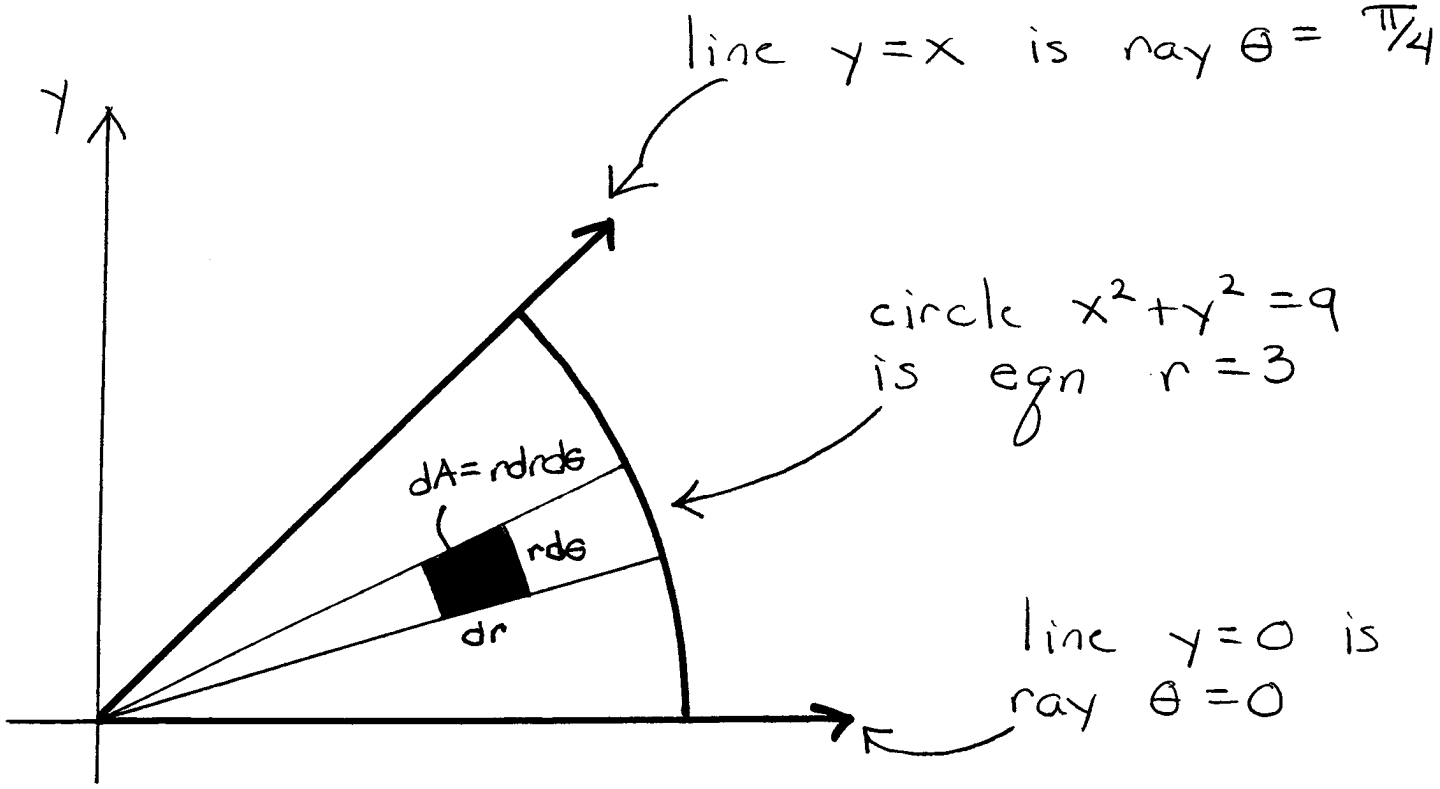
Ex. Evaluate $\iint_R y\,dA$ if R is the region in Quad. I bound between the circle $x^2+y^2=9$ & the lines $y=x$ & $y=0$.



Outer "rectangle" rotates from (thickness = $rd\theta$)
$\theta = 0$ (LL) to $\theta = \pi/4$ (UL)

Inner "rectangle" slides from (thickness = dr)
$r = 0$ (LL) to $r = 3$ (UL)

So

recall:
$y = r\sin\theta$
$dA = rdrd\theta$

$$\iint_R y\,dA = \int_0^{\pi/4}\int_0^3 \underbrace{r\sin\theta}_{y}\,\underbrace{(rdrd\theta)}_{dA}$$

$$= \int_0^{\pi/4} \sin\theta \left(\int_0^3 r^2 dr \right) d\theta$$

$$= \int_0^{\pi/4} \sin\theta \cdot \frac{1}{3} r^3 \Big|_{r=0}^{3} d\theta$$

$$= \frac{1}{3} \int_0^{\pi/4} \sin\theta \cdot (3^3 - 0^3) d\theta$$

$$= 9 \int_0^{\pi/4} \sin\theta \, d\theta$$

$$= -9 \cos\theta \Big|_{\theta=0}^{\pi/4}$$

$$= -9 \left(\cos \pi/4 - \cos 0 \right)$$

$$= -9 \left(\frac{\sqrt{2}}{2} - 1 \right)$$

$$= 9 \left(1 - \frac{\sqrt{2}}{2} \right)$$