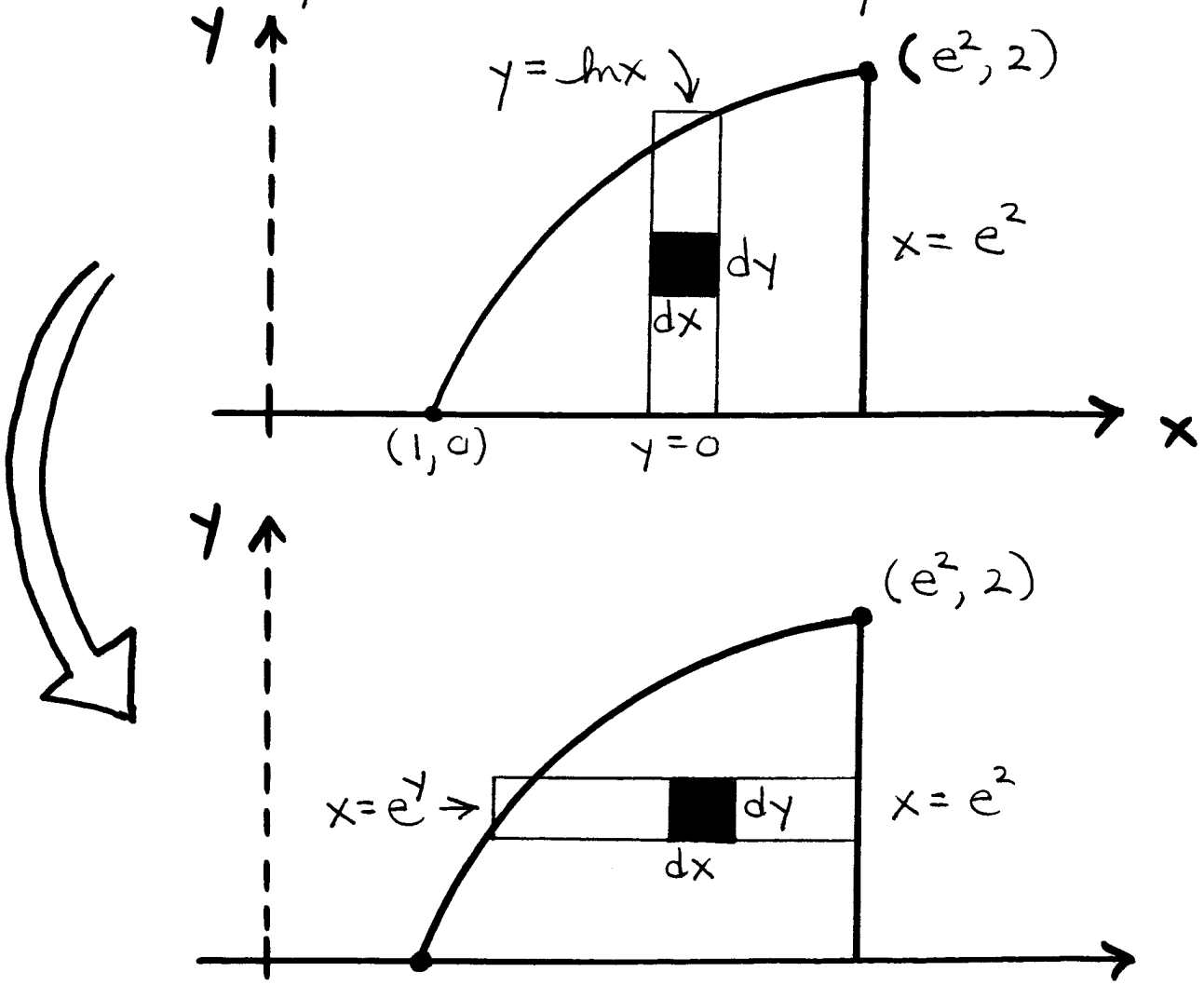
Ex. Sketch the region of integration, and change the order of integration.

$$\int_1^{e^2} \int_0^{\ln x} f(x,y)\, dy\, dx$$

Outer is dx, so x varies from $x = 1$ to $x = e^2$.

Inner is dy, so y varies from line $y = 0$ to curve $y = \ln x$.



Note:

$$y = \ln x \Rightarrow x = e^y$$

So

Outer rectangle: thickness $= dy$

LL: $y = 0$

UL: $y = 2$

Inner rectangle: thickness $= dx$

LL: $x = e^y$

UL: $x = e^2$

$$dA = dx\,dy$$

So

$$\iint_R f(x,y)\,dA$$

$$= \int_0^2 \int_{e^y}^{e^2} f(x,y)\,dx\,dy$$

So we've shown that

$$\int_1^{e^2} \int_0^{\ln x} f(x,y)\,dy\,dx$$

$$= \int_0^2 \int_{e^y}^{e^2} f(x,y)\,dx\,dy$$

Note:

$$\int_1^{e^2} \int_0^{\ln x} f\,dy\,dx \neq \int_0^{\ln x} \int_1^{e^2} f\,dx\,dy \quad !!$$