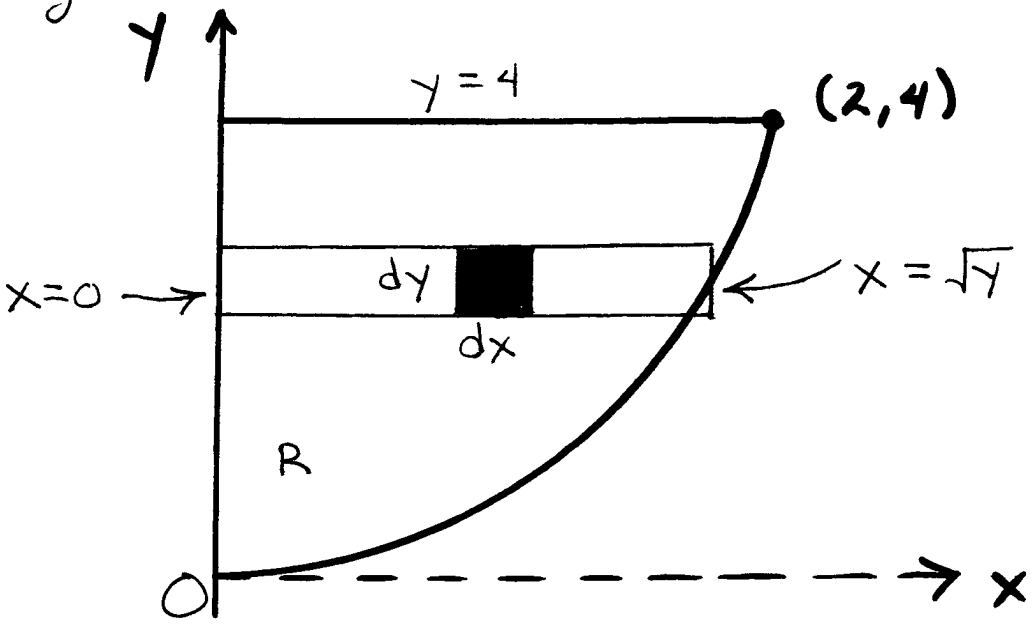
Ex. Find the volume of the solid under the surface $z = 4xe^{y^2}$ and above the region bounded by the y-axis, the curve $x = \sqrt{y}$, & the line $y = 4$.

Draw the region:



Outer rectangle: thickness = dy

LL: $y = 0$

UL: $y = 4$

Inner rectangle: thickness = dx

LL: $x = 0$

UL: $x = \sqrt{y}$

$dA = dx\,dy$

upper surface: $z = f(x,y) = 4xe^{y^2}$

$$V = \iint_R f(x,y)\,dA$$

$$V = \int_0^4 \int_0^{\sqrt{y}} 4x\, e^{y^2}\, dx\, dy$$

$$= \int_0^4 e^{y^2} \int_0^{\sqrt{y}} 4x\, dx\, dy$$

$$= \int_0^4 e^{y^2}\, 2x^2 \Big|_{x=0}^{\sqrt{y}}\, dy$$

$$= \int_0^4 2e^{y^2} \left[(\sqrt{y})^2 - 0^2\right] dy$$

$$= 2\int_0^4 y\, e^{y^2}\, dy$$

$u = y^2$

$\frac{du}{dy} = 2y$

$dy = \frac{du}{2y}$

sub

$$= 2\int y\, e^u \left(\frac{du}{2y}\right)$$

$$= \int e^u\, du$$

$$= e^u$$

back

$$= e^{y^2} \Big|_{y=0}^{4}$$

$$= e^{16} - e^0$$

$$= e^{16} - 1 \quad \text{units}^3$$