Ex. $f(x,y) = 4x^3 + y^3 - 12x - 3y$

$f_x = 12x^2 - 12 = 12(x^2-1)$ (a)

(,)

$f_y = 3y^2 - 3 = 3(y^2-1)$ (b)

CP's: Set $f_x = 0$ + $f_y = 0$:

$12(x^2-1) = 0 \Rightarrow x = -1, 1$

$3(y^2-1) = 0 \Rightarrow y = -1, 1$

CPs of f are

$(-1,-1)$, $(-1,1)$, $(1,-1)$, $(1,1)$

2nd Deriv. Test:

$f_{xx} = 24x$, $f_{yy} = 6y$

$f_{xy} = 0$

$D \equiv f_{xx} f_{yy} - f_{xy}^2$

$D = 144xy$

$D = 144xy$

CP $(-1,-1)$:

$D(-1,-1) = +144 > 0$

$f_{xx}(-1,-1) = 24(-1) = -24 < 0$

So f has a local max at CP $(-1,-1)$: $f(-1,-1) = 10$

CP $(-1,1)$:

$D(-1,1) = -144 < 0$

So f has a saddle pt @ $(-1,1)$

CP $(1,-1)$:

$D(1,-1) = -144 < 0$

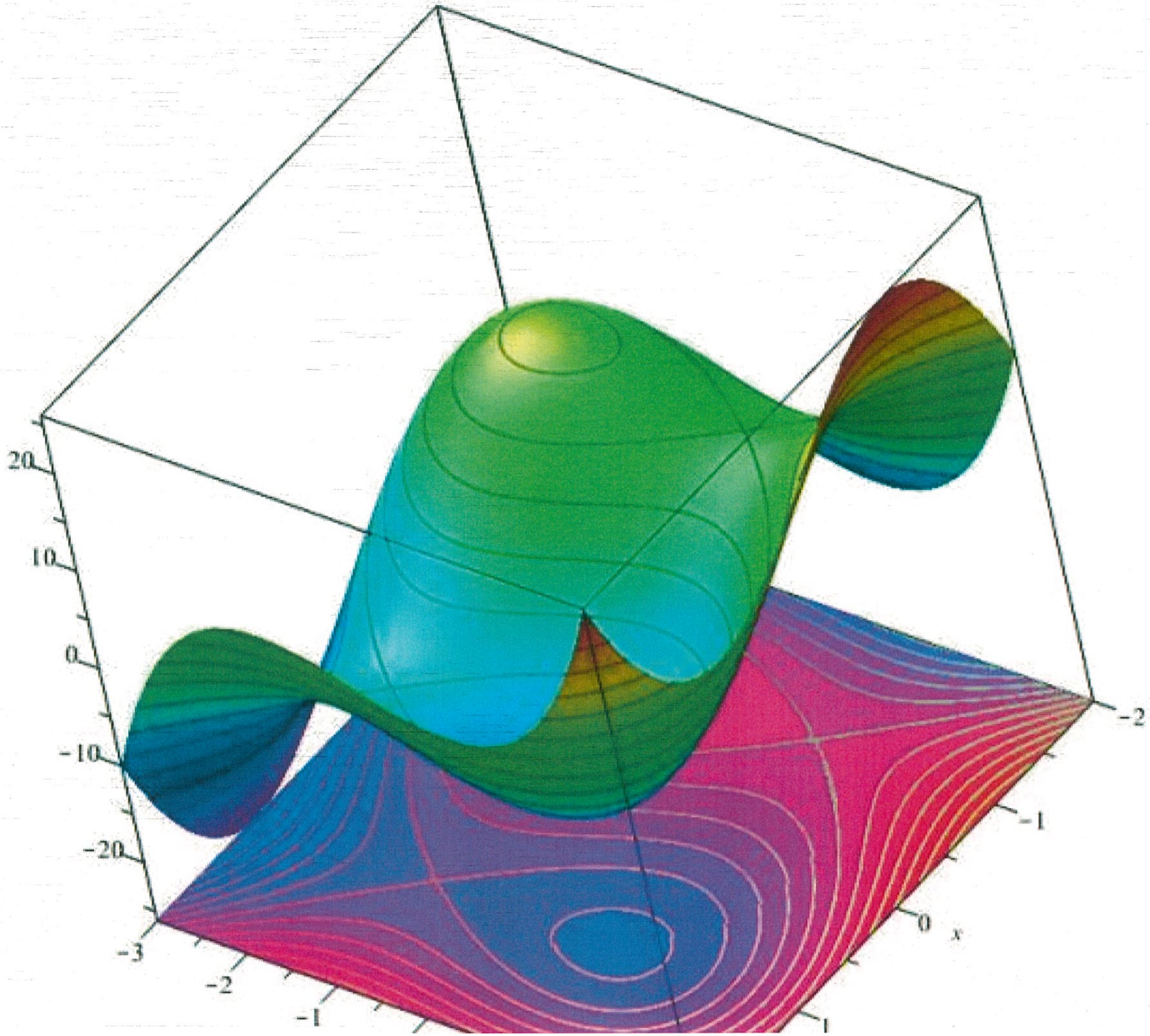
So f has saddle pt @ $(1,-1)$

CP $(1,1)$:

$D(1,1) = 144 > 0$

$f_{xx}(1,1) = 24 > 0$

So f has a local min at $(1,1)$: $f(1,1) = -10$


20
10
0
−10
−20
−3
−2
−1
−2
−1
0
x
1