MATH 203 **Partial Derivatives** **Dr. TeBeest**

First, recall that the graph of the function $z = f(x, y)$ is a **surface in** $\mathbf{R^3}$.

I. To understand the meaning of the partial derivative $f_x(a, b)$:

- Consider the cross section (trace) of the surface $z = f(x, y)$ in the plane $y = b$.
  That trace is a curve C_1 on the surface in the plane $y = b$.
- The value $f_x(a, b)$ has two meanings:
  1. **Geometric (graphical):** The value $f_x(a, b)$ gives the *slope of the tangent line to trace* C_1 *at point* (a, b).
  2. **Physical:** The value $f_x(a, b)$ gives the *rate of change of* f *with respect to* x *at point* (a, b). Specifically:

     If $f_x(a, b) > 0$, then f is *increasing* in the $+x$ direction in the plane $y = b$.

     If $f_x(a, b) < 0$, then f is *decreasing* in the $+x$ direction in the plane $y = b$.

II. To understand the meaning of the partial derivative $f_y(a, b)$:

- Consider the cross section (trace) of the surface $z = f(x, y)$ in the plane $x = a$.
  That trace is a curve C_2 on the surface in the plane $x = a$.
- The value $f_y(a, b)$ has two meanings:
  1. **Geometric (graphical):** The value $f_y(a, b)$ gives the *slope of the tangent line to trace* C_2 *at point* (a, b).
  2. **Physical:** The value $f_y(a, b)$ gives the *rate of change of* f *with respect to* y *at point* (a, b). Specifically:

     If $f_y(a, b) > 0$, then f is *increasing* in the $+y$ direction in the plane $x = a$.

     If $f_y(a, b) < 0$, then f is *decreasing* in the $+y$ direction in the plane $x = a$.