Example 6: Plot the curve that is represented parametrically by the equations

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$$x = t^3 - 4 \cdot t \quad \text{and} \quad y = t^3 - 3 \cdot t^2$$

on the interval $-2 \le t \le 4$.

```
> restart ;
> with(plots) :
> f := t -> t^3 - 4*t ;
```

$$f := t \rightarrow t^3 - 4\,t \qquad (1)$$

```
> g := t -> t^3 - 3*t^2 ;
```

$$g := t \rightarrow t^3 - 3\,t^2 \qquad (2)$$

```
> a := -2 ;
```

$$a := -2 \qquad (3)$$

```
> b := 4 ;
```

$$b := 4 \qquad (4)$$

```
> Subints := 30 ;
```

$$Subints := 30 \qquad (5)$$

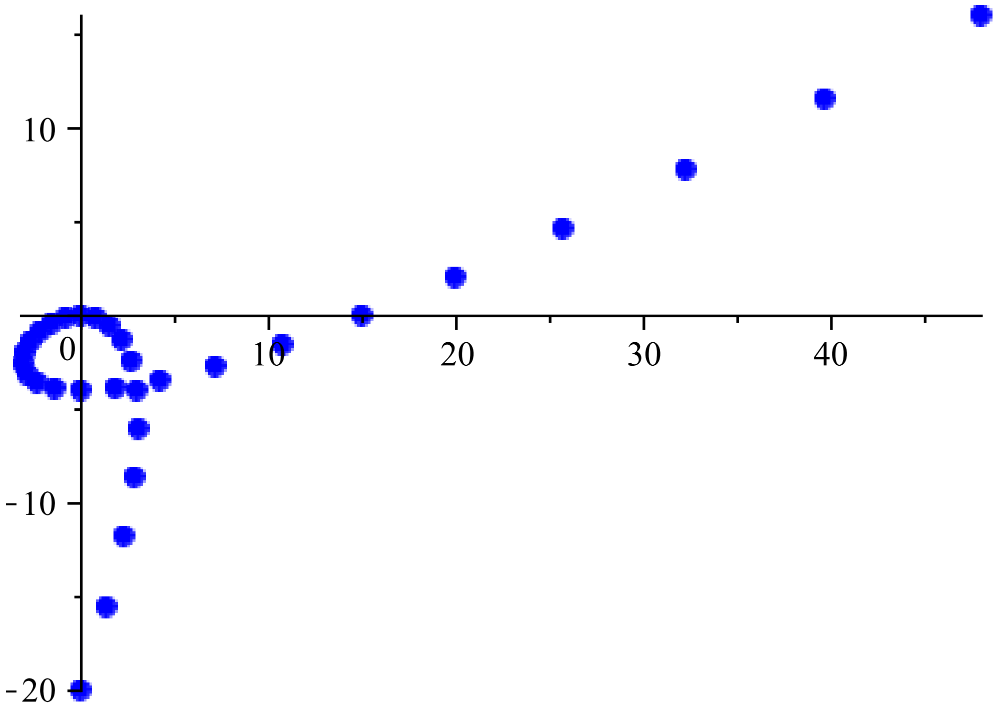
```
> h := (b-a)/Subints ;
```

$$h := \frac{1}{5} \qquad (6)$$

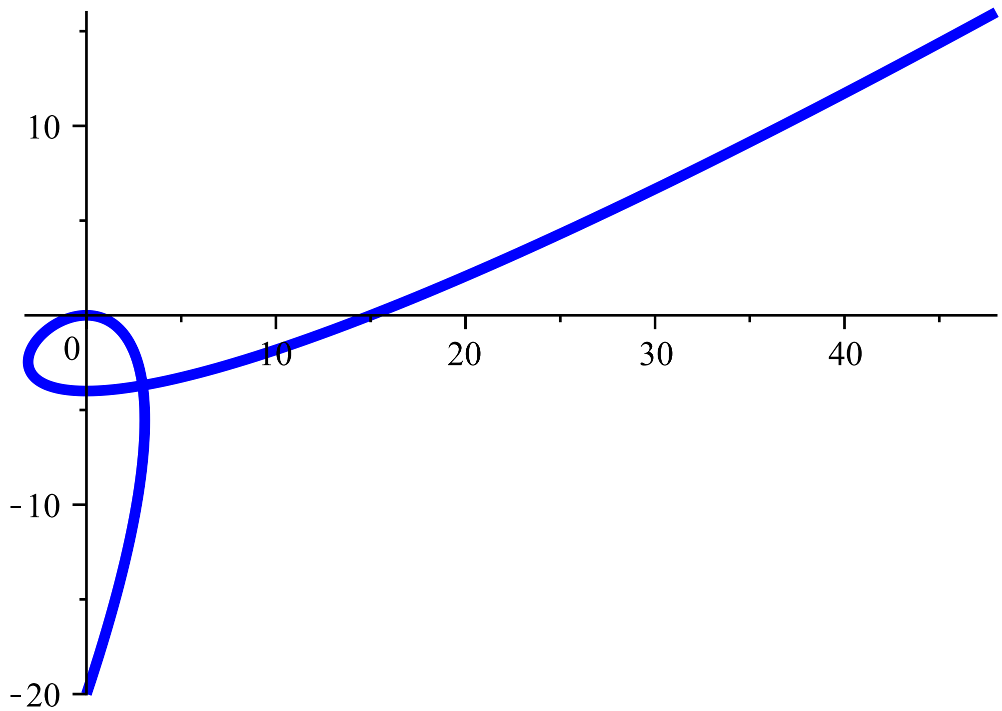
```
> printf("\n    i        t             x              y\n  ----------
  -----------------------------------\n"):
  for i from 0 to Subints do
      T[i] := a + h*i:
      X[i] := f(T[i]):
      Y[i] := g(T[i]):
      printf("  %3d  %10.5f  %12.7f  %12.7f\n", i, T[i], X[i], Y[i]
  ):
  od:

   i        t             x              y
 ---------------------------------------------
   0    -2.00000     0.0000000   -20.0000000
   1    -1.80000     1.3680000   -15.5520000
   2    -1.60000     2.3040000   -11.7760000
   3    -1.40000     2.8560000    -8.6240000
   4    -1.20000     3.0720000    -6.0480000
   5    -1.00000     3.0000000    -4.0000000
   6    -0.80000     2.6880000    -2.4320000
```

7	-0.60000	2.1840000	-1.2960000
8	-0.40000	1.5360000	-0.5440000
9	-0.20000	0.7920000	-0.1280000
10	0.00000	0.0000000	0.0000000
11	0.20000	-0.7920000	-0.1120000
12	0.40000	-1.5360000	-0.4160000
13	0.60000	-2.1840000	-0.8640000
14	0.80000	-2.6880000	-1.4080000
15	1.00000	-3.0000000	-2.0000000
16	1.20000	-3.0720000	-2.5920000
17	1.40000	-2.8560000	-3.1360000
18	1.60000	-2.3040000	-3.5840000
19	1.80000	-1.3680000	-3.8880000
20	2.00000	0.0000000	-4.0000000
21	2.20000	1.8480000	-3.8720000
22	2.40000	4.2240000	-3.4560000
23	2.60000	7.1760000	-2.7040000
24	2.80000	10.7520000	-1.5680000
25	3.00000	15.0000000	0.0000000
26	3.20000	19.9680000	2.0480000
27	3.40000	25.7040000	4.6240000
28	3.60000	32.2560000	7.7760000
29	3.80000	39.6720000	11.5520000
30	4.00000	48.0000000	16.0000000

```
> plot( [[ f(T[k]), g(T[k]) ]$k = 0 .. Subints ], style=point,
  symbol=solidcircle, symbolsize=16, color=blue, scaling =
  constrained, view=[-3.2 .. 48,-20..16] ) ;
```




```
> plot( [ f(t), g(t), t = a .. b ], color = blue, thickness = 4,
  scaling = constrained, view=[-3.2 .. 48,-20..16] );
```



Create the animation to see the curve's growth and direction:

```
> animatecurve([ f(t),g(t), t = a .. b], frames=50, color=blue,
  thickness=4, numpoints=200, scaling = constrained, view=[-3.2 ..
  48,-20..16] );
```

Determine the two values of t (call them t_1 and t_2) at which the curve intersects itself. This occurs when

$$x(t_1) = x(t_2) \quad \text{AND} \quad y(t_1) = y(t_2).$$

That is, when

$$f(t_1) = f(t_2) \quad \text{AND} \quad g(t_1) = g(t_2).$$

```
> eq1 :=  f(t1) = f(t2) ;   #  this defines the first equation as
  "eq1"
```

(7)

$$eq1 := t1^3 - 4\,t1 = t2^3 - 4\,t2 \quad (7)$$

```
> eq2 :=  g(t1) = g(t2) ;  #  this define the second equation as
  "eq2"
```

$$eq2 := t1^3 - 3\,t1^2 = t2^3 - 3\,t2^2 \quad (8)$$

Now solve the two equations for t_1 and t_2 and call the solution "soln":

```
> soln := evalf( solve( [eq1,eq2] , [t1,t2] ) );
```

$$soln := [[t1 = t2, t2 = t2], [t1 = -0.966326496, t2 = 2.299659829]] \quad (9)$$

Store the second solution and call it times:

```
> times := soln[2] ;
```

$$times := [t1 = -0.966326496, t2 = 2.299659829] \quad (10)$$

Assign the values to t_1 and t_2 :

```
> assign(times) ;
```

Check to see that the two times t_1 and t_2 are stored properly:

```
> t1 ;
```

$$-0.966326496 \quad (11)$$

```
> t2 ;
```

$$2.299659829 \quad (12)$$

Evaluate $f(t_1)$ and $f(t_2)$ to show that we get the same x coordinate:

```
> f(t1) ;
```

$$2.962962964 \quad (13)$$

```
> f(t2) ;
```

$$2.962962964 \quad (14)$$

Evaluate $g(t_1)$ and $g(t_2)$ to show that we get the same y coordinate:

```
> g(t1) ;
```

$$-3.703703711 \quad (15)$$

```
> g(t2) ;
```

$$-3.70370371 \quad (16)$$

So the parametric curve crosses itself at the point $(2.962962964, -3.703703711)$ when $t_1 = -0.966326496$ and $t_2 = 2.299659829$.