10.1 Parametric Equations of Curves in 2-D

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Often we cannot express the formula of a curve in the xy plane in the form

$$y = f(x) .$$

Let (x, y) denote an arbitrary point on a curve in the $x\,y$-plane.
Often we can express the point's x and y coordinates in the form of two equations---

$$x = f(t) \quad \text{and} \quad y = g(t) ,$$

where t is called a **parameter**. For example, a specific value like t_0 describes the point

$$(x_0, y_0) = (f(t_0), g(t_0))$$

on the curve. For each value of parameter t in the domain, we can determe the x and y coordinates of all points on the curve and plot the points. Such a curve is called a **parametric curve**.

Example 1: Plot the parametric curve represented parametrically by the equations

$$x = \sin(3\,t) \quad \text{and} \quad y = \sin(4\,t)$$

on the interval $0 \le t \le 2\,\pi$.

```
> restart ;
> with(plots) :
> f := t -> sin(3*t) ;
```

$$f := t \to \sin(3\,t) \tag{1}$$

```
> g := t -> sin(4*t) ;
```

$$g := t \to \sin(4\,t) \tag{2}$$

```
> a := 0 ;
```

$$a := 0 \tag{3}$$

```
> b := 2*Pi ;
```

$$b := 2\,\pi \tag{4}$$

```
> Subints := 30 ;
```

$$Subints := 30 \tag{5}$$

```
> h := (b-a)/Subints ;
```

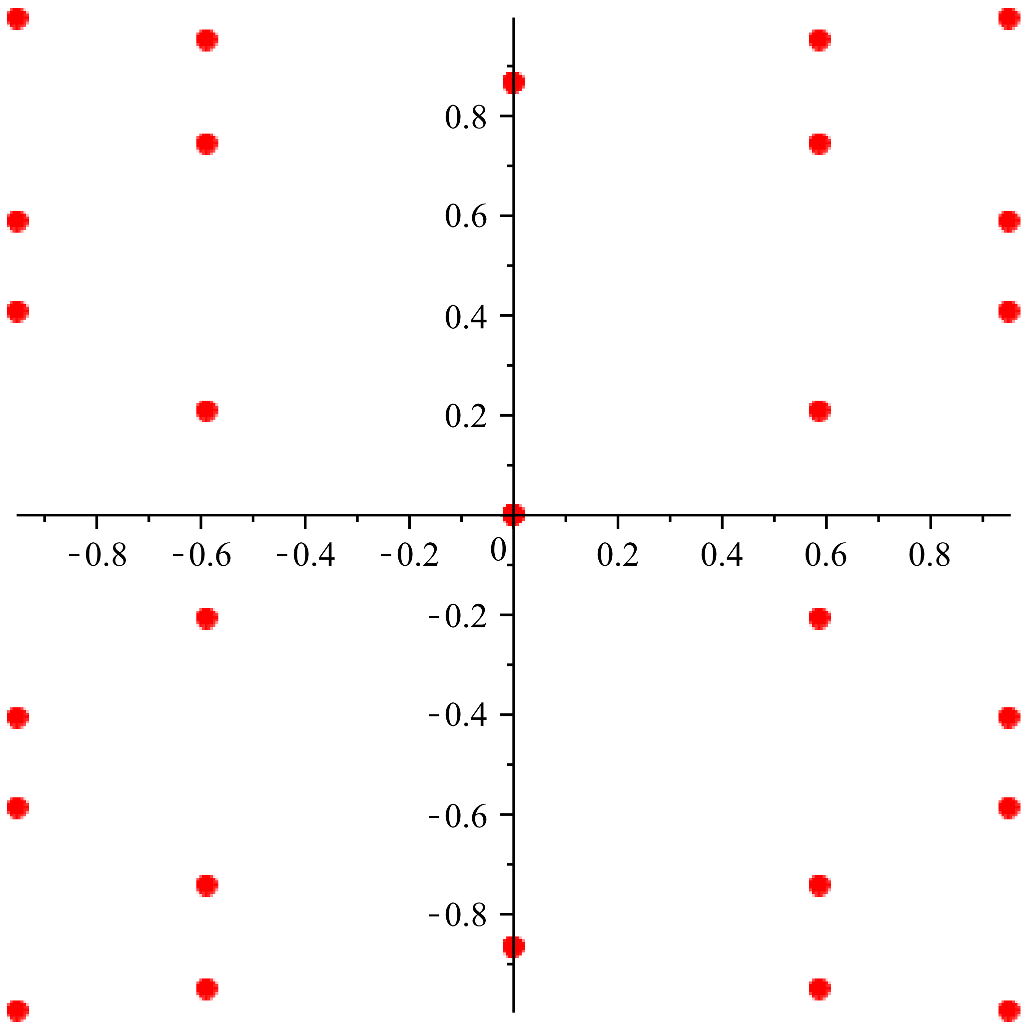
$$h := \frac{1}{15}\,\pi \tag{6}$$

```
> printf("\n    i        t             x              y\n   ----------
  -----------------------------------\n"):
```

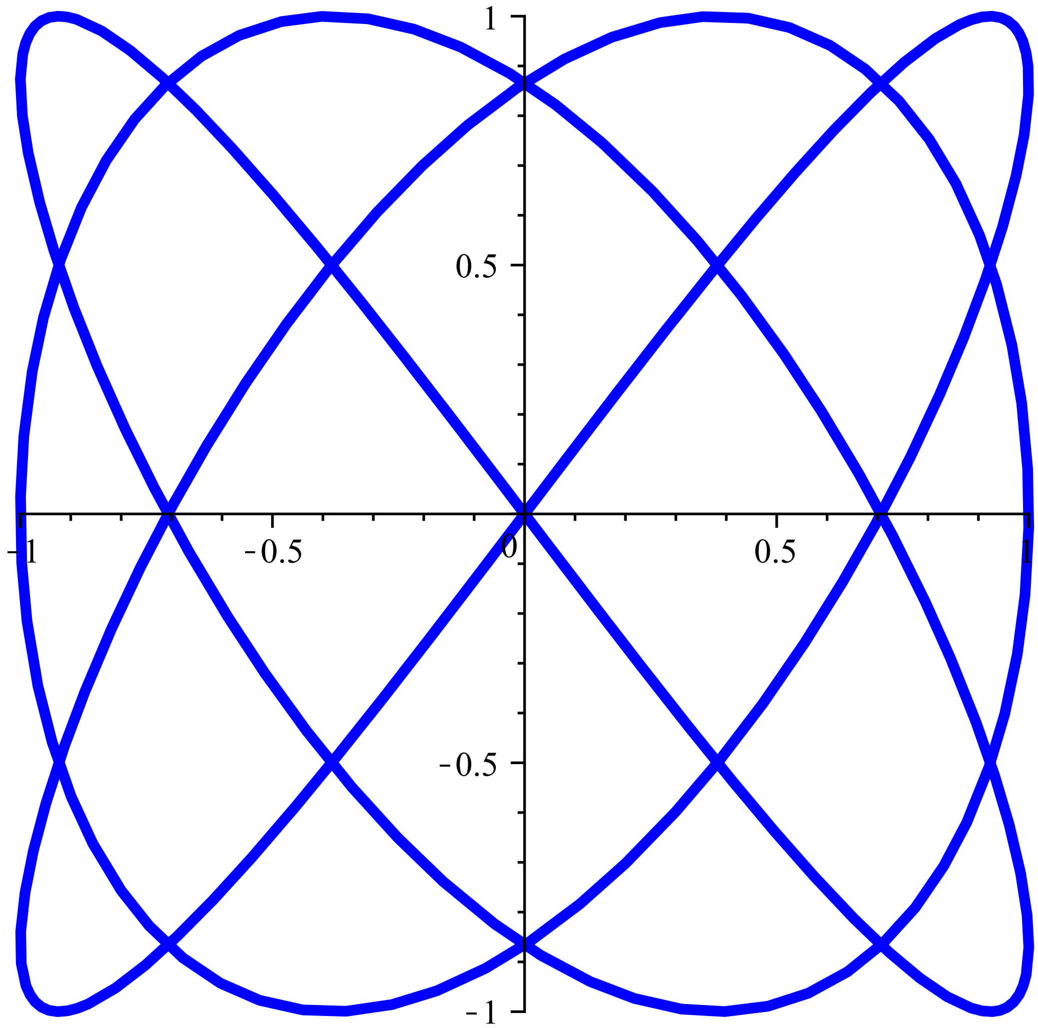
```
for i from 0 to Subints do
    T[i] := a + h*i:
    X[i] := f(T[i]):
    Y[i] := g(T[i]):
    printf("  %3d  %10.5f  %12.7f  %12.7f\n", i, T[i], X[i], Y[i]
):
od:
```

```
  i        t            x             y
-------------------------------------------
  0     0.00000     0.0000000     0.0000000
  1     0.20944     0.5877853     0.7431448
  2     0.41888     0.9510565     0.9945219
  3     0.62832     0.9510565     0.5877853
  4     0.83776     0.5877853    -0.2079117
  5     1.04720     0.0000000    -0.8660254
  6     1.25664    -0.5877853    -0.9510565
  7     1.46608    -0.9510565    -0.4067366
  8     1.67552    -0.9510565     0.4067366
  9     1.88496    -0.5877853     0.9510565
 10     2.09440     0.0000000     0.8660254
 11     2.30383     0.5877853     0.2079117
 12     2.51327     0.9510565    -0.5877853
 13     2.72271     0.9510565    -0.9945219
 14     2.93215     0.5877853    -0.7431448
 15     3.14159     0.0000000     0.0000000
 16     3.35103    -0.5877853     0.7431448
 17     3.56047    -0.9510565     0.9945219
 18     3.76991    -0.9510565     0.5877853
 19     3.97935    -0.5877853    -0.2079117
 20     4.18879     0.0000000    -0.8660254
 21     4.39823     0.5877853    -0.9510565
 22     4.60767     0.9510565    -0.4067366
 23     4.81711     0.9510565     0.4067366
 24     5.02655     0.5877853     0.9510565
 25     5.23599     0.0000000     0.8660254
 26     5.44543    -0.5877853     0.2079117
 27     5.65487    -0.9510565    -0.5877853
 28     5.86431    -0.9510565    -0.9945219
 29     6.07375    -0.5877853    -0.7431448
 30     6.28319     0.0000000     0.0000000
```

```
> plot( [[ f(T[k]), g(T[k]) ]$k = 0 .. Subints ], style=point,
  symbol=solidcircle, symbolsize=16, color=red);
```

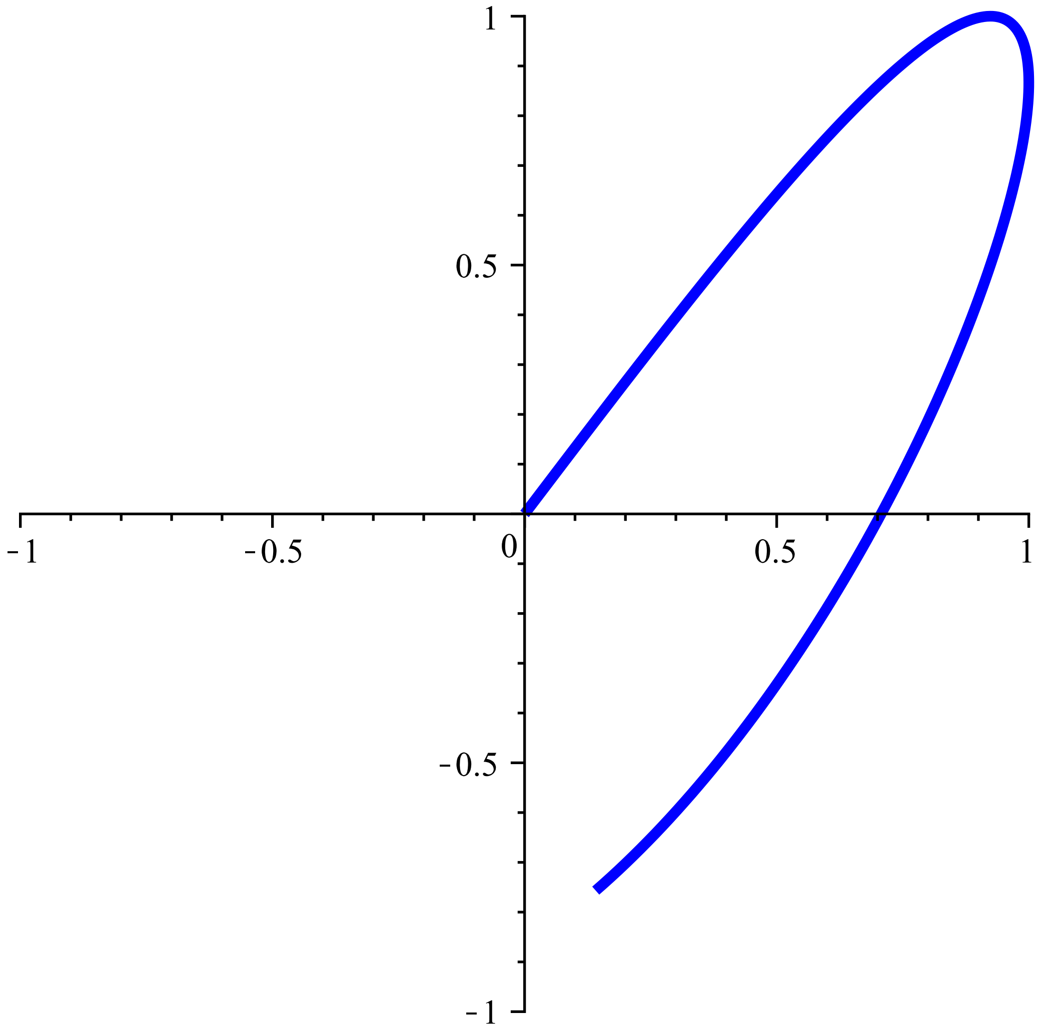



```
> plot( [ f(t), g(t), t = a..b ],color=blue, thickness=4 );
```

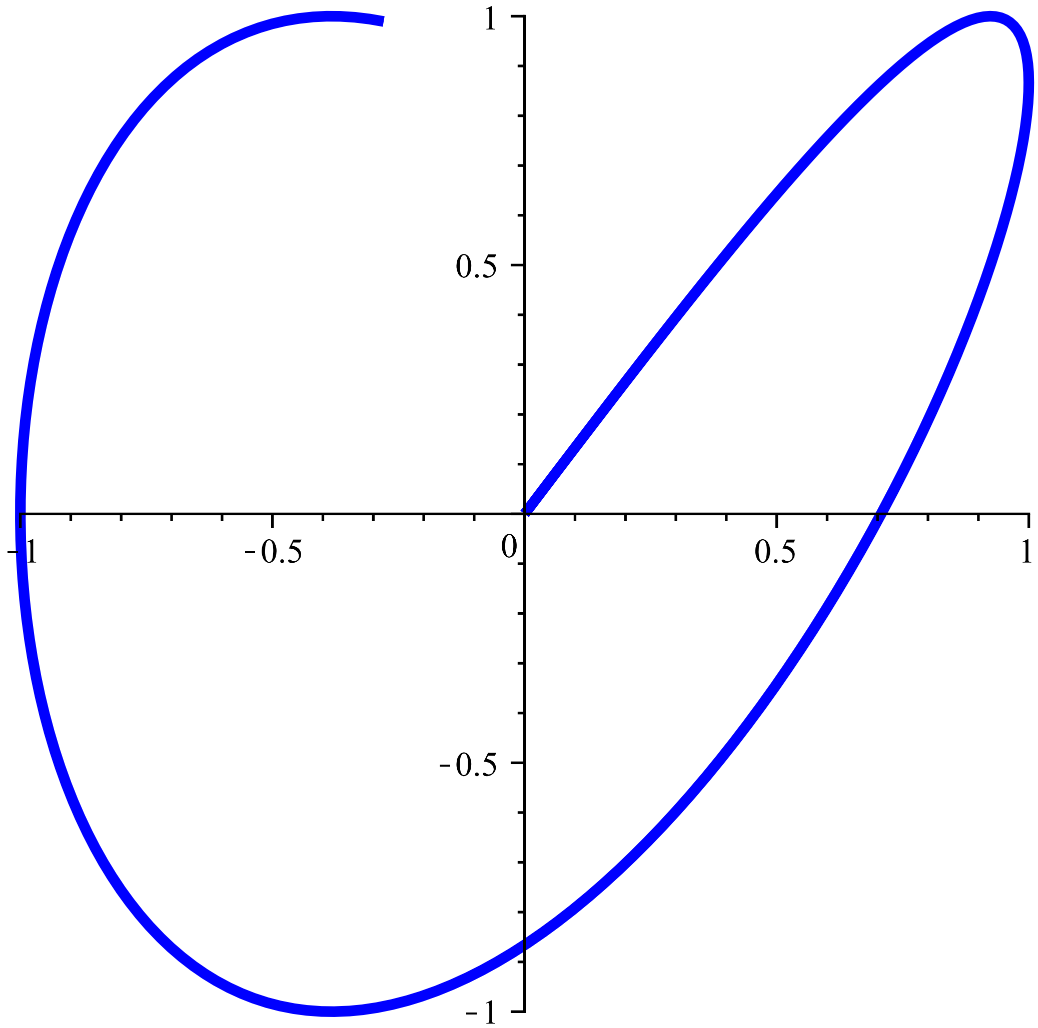


To see the direction of the curve, plot it on shorter time intervals, and watch the curve's growth as t increases.

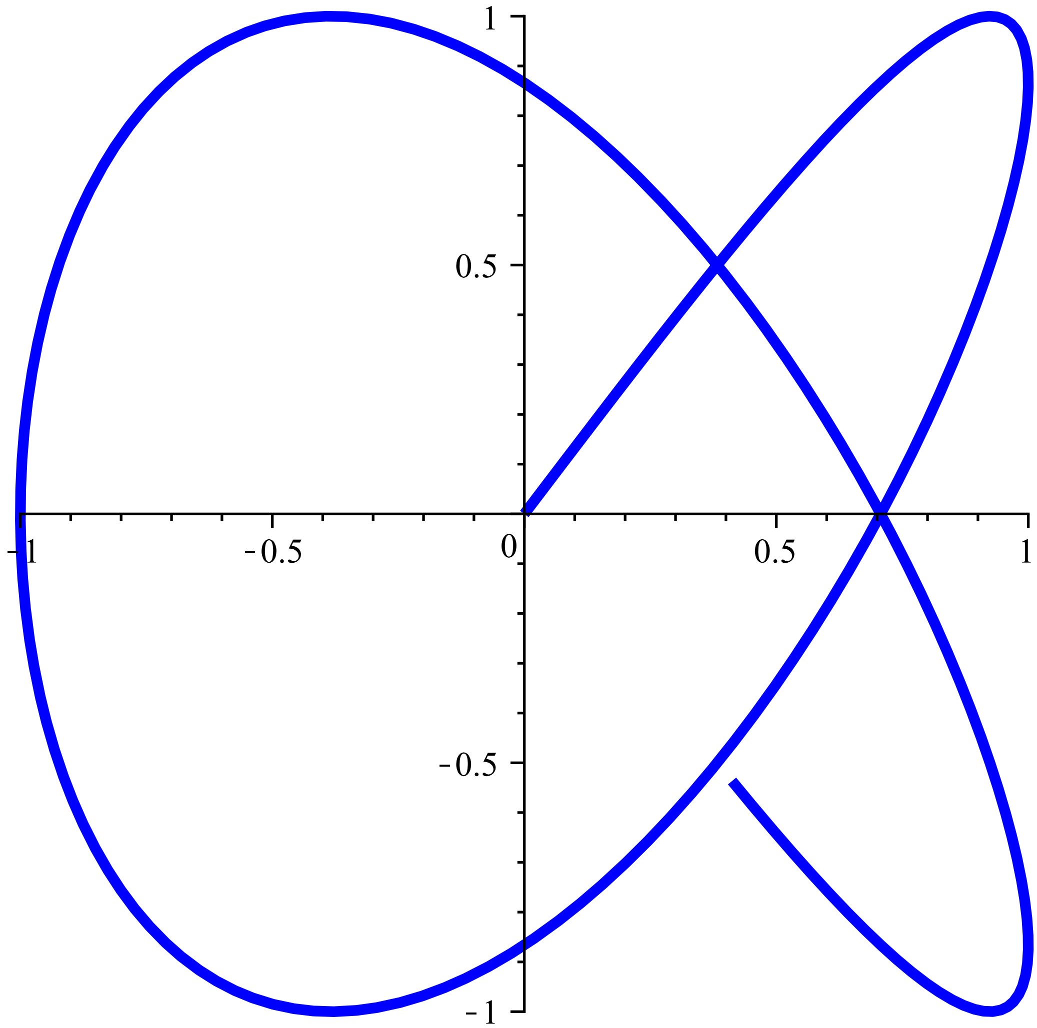
```
> plot( [ f(t), g(t), t = 0 .. 1 ],color=blue, thickness=4, view=
  [-1..1,-1..1] );
```



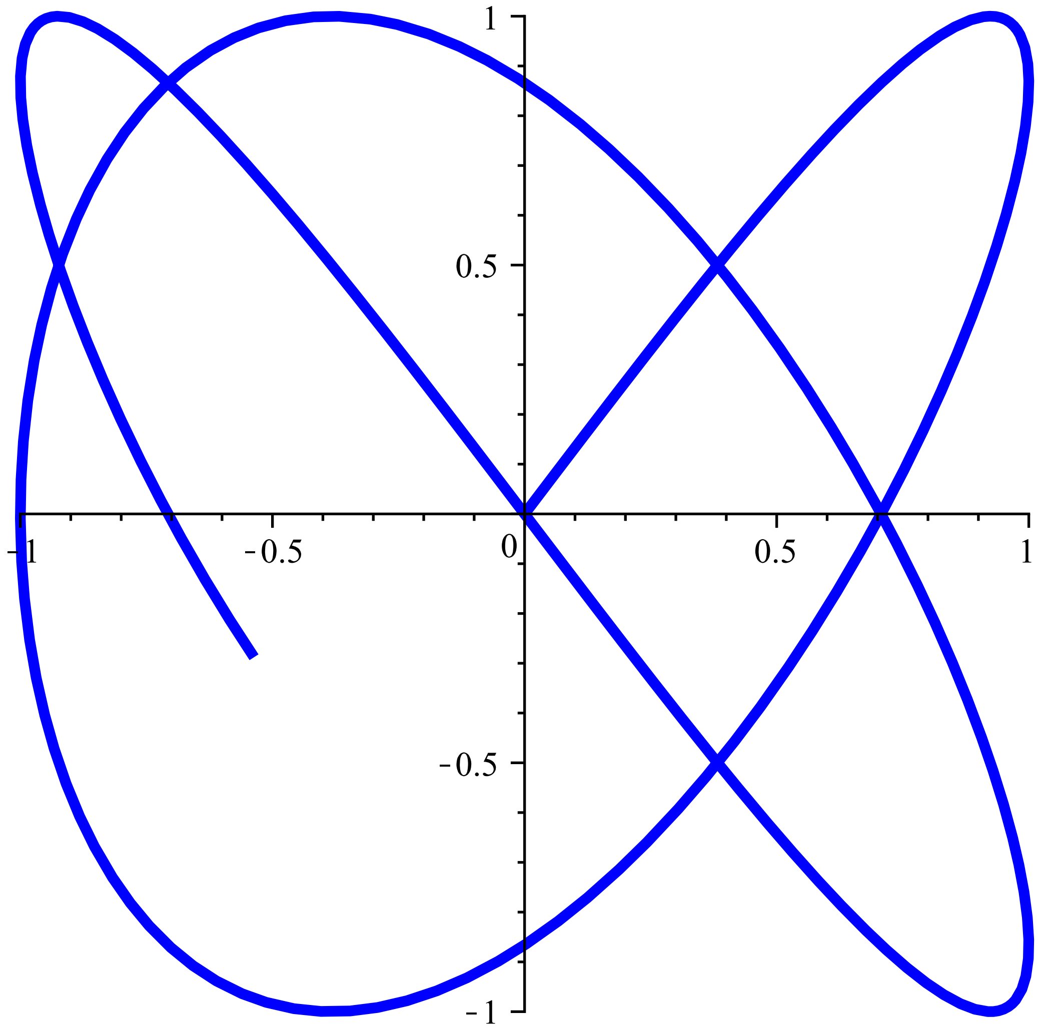

```
> plot( [ f(t), g(t), t = 0 .. 2 ],color=blue, thickness=4, view=
  [-1..1,-1..1] );
```



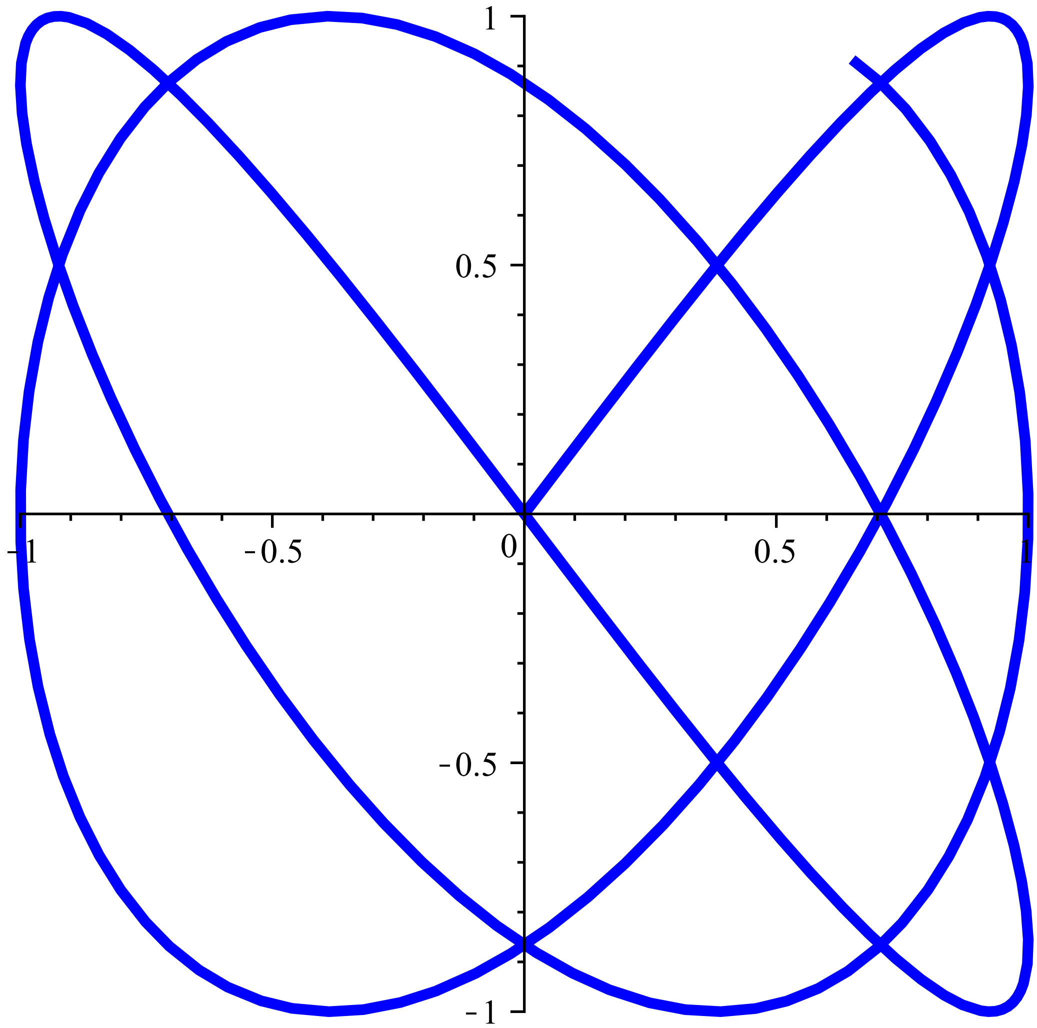

```
> plot( [ f(t), g(t), t = 0 .. 3 ],color=blue, thickness=4, view=
  [-1..1,-1..1] );
```



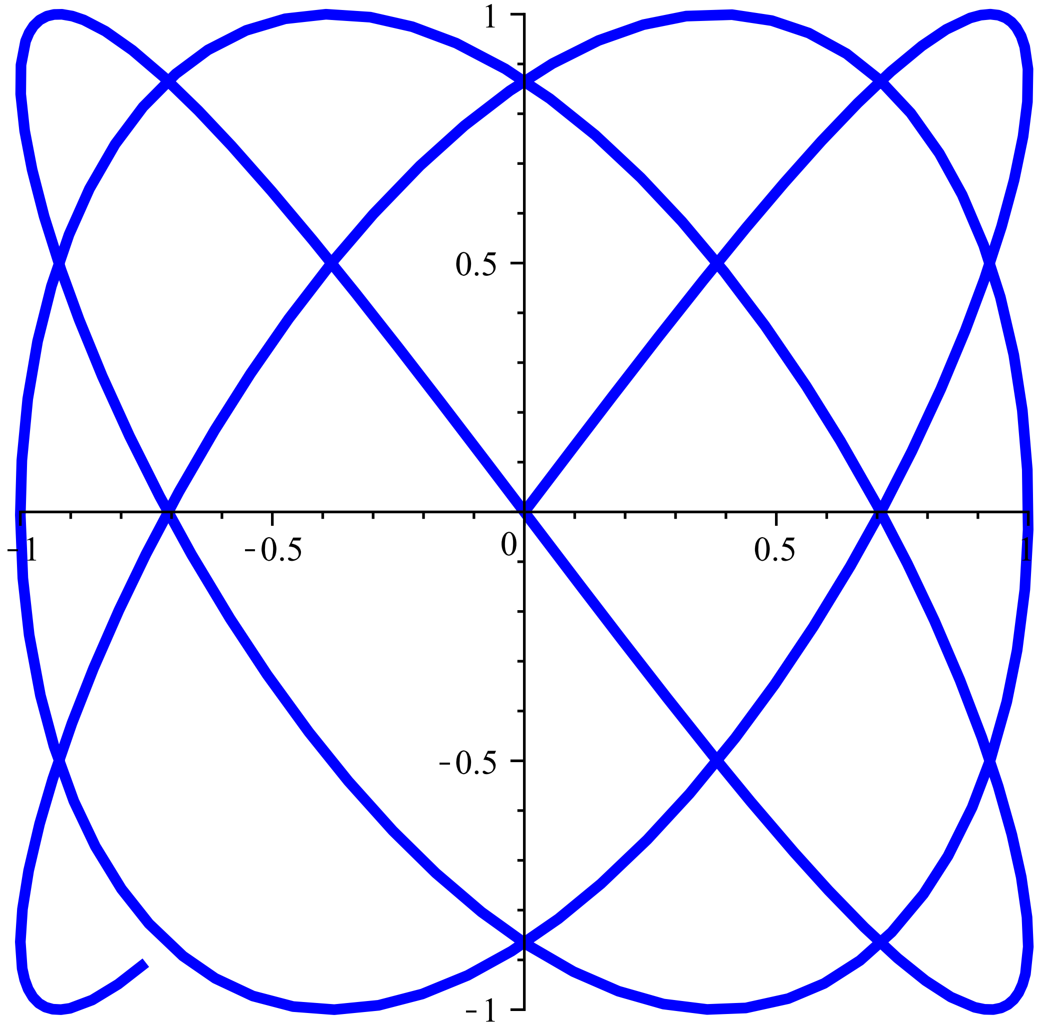

```
> plot( [ f(t), g(t), t = 0 .. 4 ],color=blue, thickness=4, view=
  [-1..1,-1..1] );
```



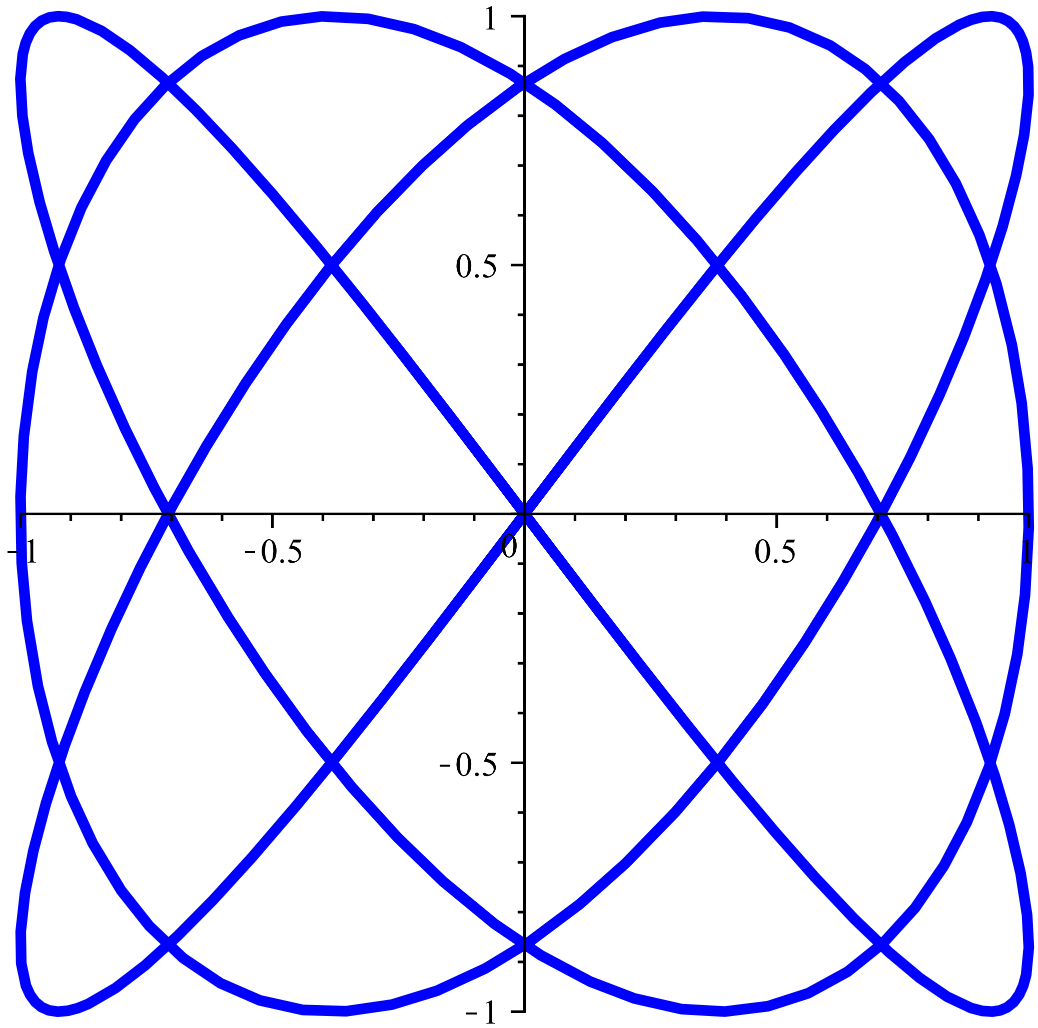

```
> plot( [ f(t), g(t), t = 0 .. 5 ],color=blue, thickness=4, view=
  [-1..1,-1..1] );
```




```
> plot( [ f(t), g(t), t = 0 .. 6 ],color=blue, thickness=4, view=
  [-1..1,-1..1] );
```




```
> plot( [ f(t), g(t), t = a .. b ],color=blue, thickness=4, view=
  [-1..1,-1..1] );
```



Notice that this curve will begin to trace over itself as we plot it for values of $t > 2\pi$ **.**

Create the animation to see the parametric curve and its direction:

```
> animatecurve([ f(t),g(t), t = a .. b], frames=100, color=blue,
  thickness=4, numpoints=200, view=[-1..1,-1..1], scaling =
  constrained );
>
```