Chapter 9: Transient Behavior in the Time Domain

9.1 Design a circuit using reasonable values for the components that is capable of providing a time delay of 100 ms to a digital signal. State all assumptions.

9.2C Starting with the integral equation for an *RC* integrator provided in this chapter

$$v_c(t) = \frac{1}{C}\int_0^t i_C(t)\,dt + v_C(0)$$

determine the expression for the output voltage if

$$i_C(t) = 5e^{-\alpha t}u(t)$$

Then, plot both the input and output versus time on the same set of axes for each of the given cases:

$$\alpha = \frac{1}{RC},\ \alpha = \frac{5}{RC},\ \alpha = \frac{1}{5RC}$$

9.3C Starting with the differential equation for an *RC* differentiator provided in this chapter

$$\frac{dv_R}{dt} + \frac{v_R}{RC} - \frac{dv_i}{dt} = 0$$

determine the expression for the output voltage if

$$v_i(t) = 4.5\left(1 - e^{-\alpha t}\right)u(t)$$

Then, plot both the input and output versus time on the same set of axes for each of the given cases:

$$\alpha = \frac{1}{RC},\ \alpha = \frac{5}{RC},\ \alpha = \frac{1}{5RC}$$

9.4SC Repeat Problem 9.3 for the following ramp signal:

$$v_i(t) = \alpha t u(t)$$

9.5. Verify that the results given for Case 1 for the general *RC* circuit analysis given in this chapter are correct by reanalyzing the Case 1 circuit. Do not merely use the expression provided for the general *RC* circuit but repeat the analysis beginning with

$$A + Be^{-\frac{t}{\tau}}.$$

9.6 Repeat Problem 9.5 for Case 2.
9.7 Repeat Problem 9.5 for Case 3.
9.8 Repeat Problem 9.5 for Case 4.
9.9 Determine the expression for the voltage across the capacitor, v_C, and the current through the load resistor, i, for the pre-emphasis circuit given in Figure 1.

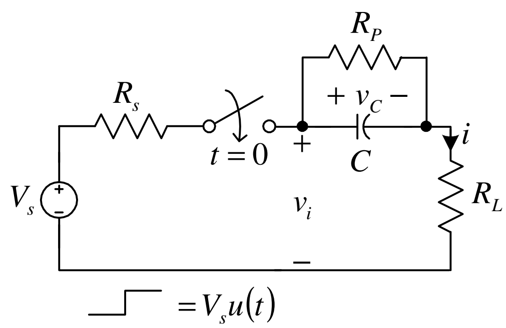

Figure 1

9.10S For the general *RC* circuit example given in this chapter, repeat the entire analysis but replace V_s and R_s with its Norton equivalent; that is, replace the voltage source and corresponding resistance with an independent current source of value V_o/R_s in parallel with a resistance R_s. Do not used source transformations but repeat the analysis using this current source. A current source is sometimes approximated by placing a high resistance in series with a high-voltage source. Why does this approximate a current source?
9.11C Repeat the analysis for the static charge buildup discussion in this chapter but assume that a resistor, R_p, is temporarily placed in shunt with the capacitor at $t = t_1 + (t_2 - t_1)/2$. It is removed from the circuit at $t = t_2$ as shown in Figure 2. For the numerical analysis, let $R_p = R$, $R_p = 10R$, and $R_p = R/10$ for all three specific values of C given.

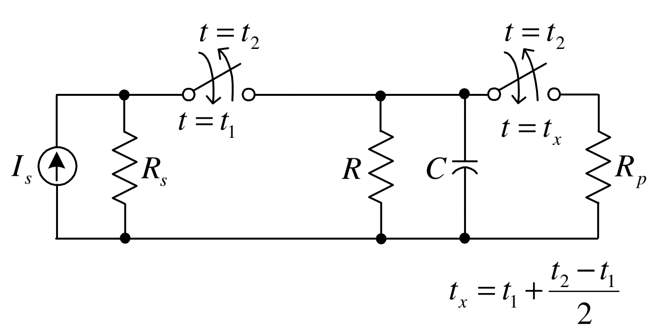

Figure 2

9.12 Determine for $t > 0$, $v_o(t)$, $i(t)$, and the energy dissipated in the 1 kΩ resistor from $t = 0$ to 2τ and from $t = 2\tau$ to 10τ for the circuit given in Figure 3.

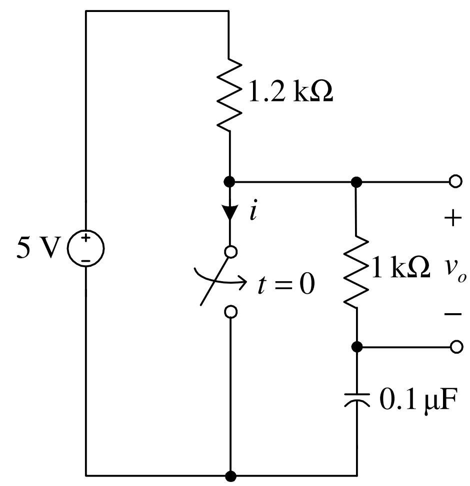

Figure 3

9.13 For $t > 0$, determine $v_o(t)$, $i(t)$, and the maximum energy that can be dissipated in both of the 2.2 kΩ resistors for the circuit given in Figure 4.

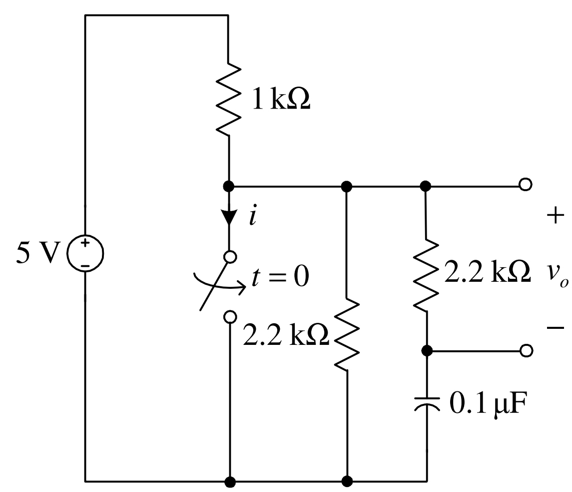

Figure 4

9.14 Determine for $t > 0$ the current $i(t)$, the voltage across each of the capacitors, and the energy released from both capacitors from $t = 0$ to two time constants for the circuit given in Figure 5.

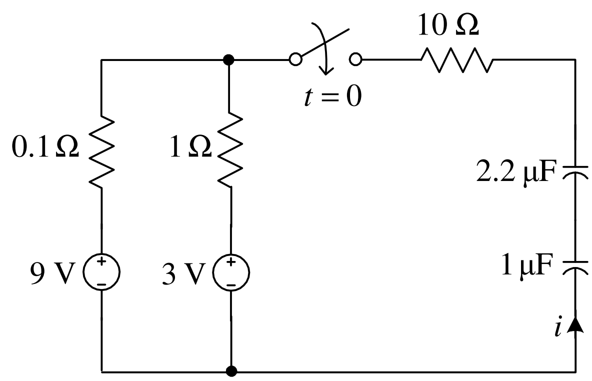

Figure 5

9.15 For the circuit given in Figure 6, determine the voltage across each of the capacitors for $t \geq 0$. Hint: first, determine the current through the net capacitance and the 220 Ω resistor. Second, use this current expression in the integral definition for the capacitor voltage for each capacitor:

$$v_c(t) = \frac{1}{C}\int_0^t i(t)\,dt + v_c(0)$$

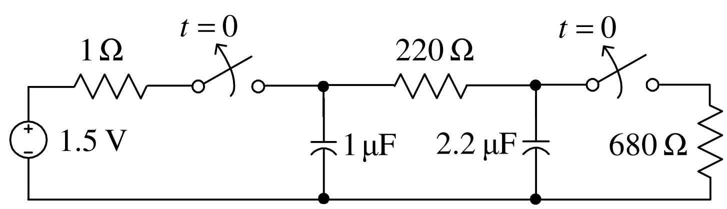

Figure 6

9.16 For the circuit given in Figure 7, determine $i(t)$ for $t > 0$.

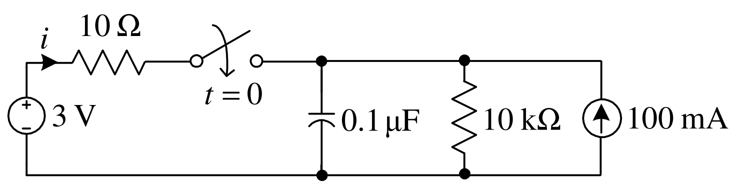

Figure 7

9.17 For the circuit given in Figure 8, determine the time required for the voltage $v_c(t)$ to rise to 6 V.

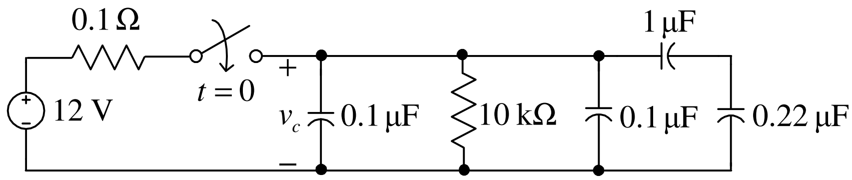

Figure 8

9.18 For the circuit given in Figure 9, determine $i(t)$ for $t > 0$.

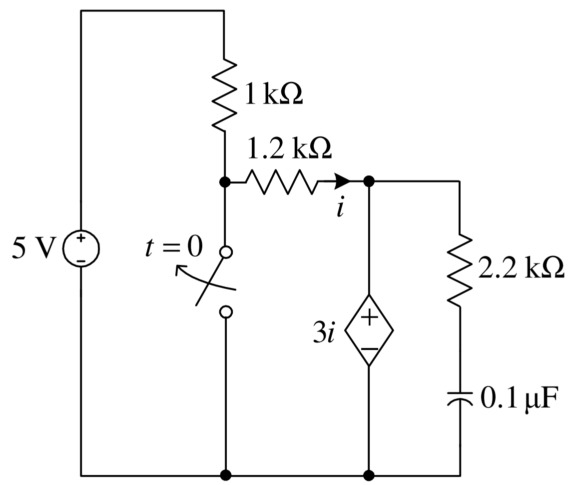

Figure 9

9.19 For the given *RC* gate in Figure 10, determine the voltage across the load resistor, R_L, for $t > 0$ if

a) $v_1(t) = 5u(t),\ v_2(t) = 5u(t - 4\tau)$

b) $v_1(t) = 5u(t - 4\tau),\ v_2(t) = 5u(t)$

c) $v_1(t) = 5u(t),\ v_2(t) = -5u(t - 4\tau)$

d) $v_1(t) = 5u(t - 4\tau),\ v_2(t) = -5u(t)$

where τ is the time constant for the system. What is this time constant in terms of R, C, and R_L?

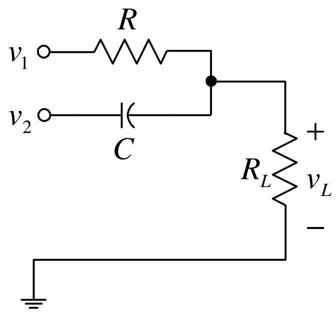

Figure 10

9.20S Can the state of the internal gates for a digital memory element change more than once during a clock cycle? Explain.

9.21 Starting with the general exponential solution, determine directly the following expression for the voltage across the resistor of a series *RL* circuit to a step voltage input by determining the initial and final voltage across the resistor:

$$v_R(t) = i_L(t)R = V_s\left(1 - e^{-\frac{t}{\tau}}\right)u(t) \quad \text{where } \tau = \frac{L}{R}$$

9.22 Starting with the general exponential solution, determine directly the following expression for the voltage across the inductor of a series *RL* circuit to a step voltage input by determining the initial and final voltage across the inductor directly:

$$v_L(t) = V_s e^{-\frac{t}{\tau}} \quad \text{if } t > 0$$

9.23 For a step current source (a current source that turns on at $t = 0$) of strength I in parallel with a parallel *RC* circuit, verify the following expressions for the voltage across the R and C and the respective currents through the R and C:

$$v(t) = IR\left(1 - e^{-\frac{t}{RC}}\right)u(t),\ i_R(t) = I\left(1 - e^{-\frac{t}{RC}}\right)u(t),\ i_C(t) = Ie^{-\frac{t}{RC}}u(t)$$

Clearly state all assumptions.

9.24 For a step current source (a current source that turns on at $t = 0$) of strength I in parallel with a parallel *RL* circuit, verify that the following expressions for the voltage across the R and L and the respective currents through the R and L:

$$v(t) = IR\mathrm{e}^{-\frac{Rt}{L}}u(t),\ i_R(t) = Ie^{-\frac{Rt}{L}}u(t),\ i_L(t) = I\left(1 - e^{-\frac{Rt}{L}}\right)u(t)$$

Clearly state all assumptions.

9.25 For a current signal source in parallel with a parallel *RC* circuit, repeat the analytical analysis given in the integrator and differentiator discussions provided in the chapter.

9.26 For a current signal source in parallel with a parallel *RL* circuit, repeat the analytical analysis given in the integrator and differentiator discussions provided in the chapter.

9.27 Determine $i(t)$, $v_R(t)$, and the energy stored in the inductor from $t = 0$ to 5τ for the circuit given in Figure 11.

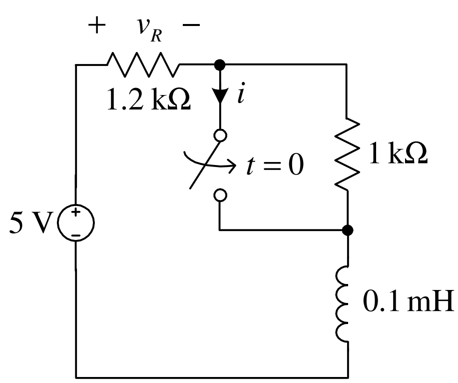

Figure 11

9.28 Determine $i(t)$ and the energy dissipated by the 1 kΩ resistor between 0.1 and 1 ns for the two-supply circuit given in Figure 12. Determine the energy delivered by the current supply over this same time range.

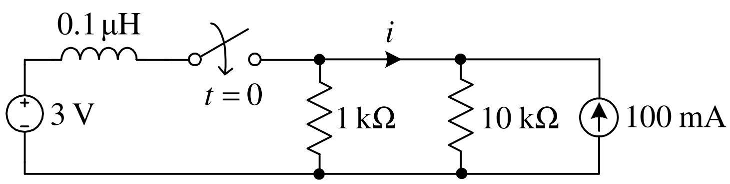

Figure 12

9.29 For the circuit given in Figure 13, determine the voltage across the inductor and the voltage across the 680 Ω resistor for $t > 0$ (where τ corresponds to the time constant of the circuit before the second switching). Determine the net energy stored by the inductor between $t = 0$ and 3τ.

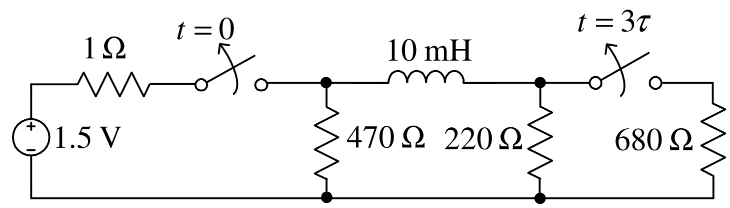

Figure 13

9.30 For the two-inductor circuit given in Figure 14, determine $v_L(t)$ and $i(t)$. After three time constants determine the energy dissipated by the 10 Ω resistor. After five time constants determine the energy dissipated by the 10 Ω resistor.

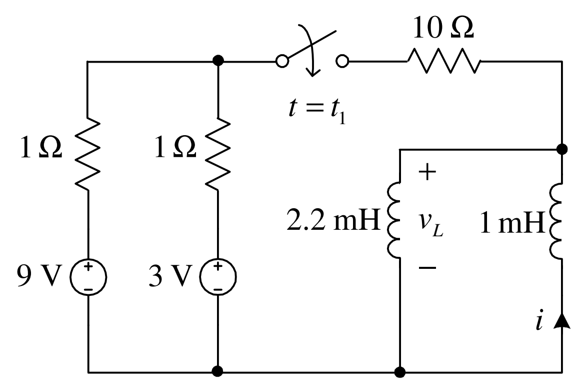

Figure 14

9.31 The switch has been closed a long time before opening at $t = 0$ for the circuit shown in Figure 15. Find the voltage $v_x(t)$ for $t > 0$. The method for determining the equivalent resistance for an *RL* circuit with a dependent supply is identical to that for an *RC* circuit.

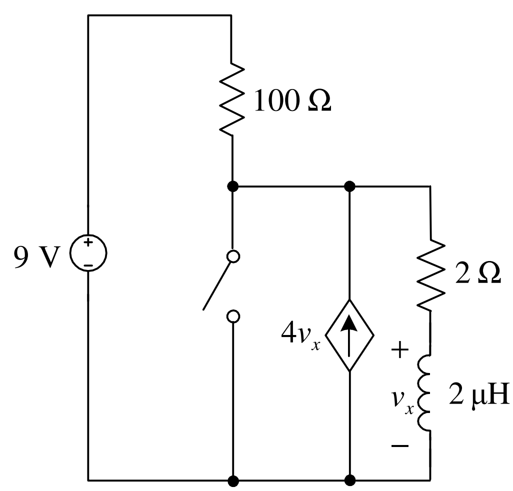

Figure 15

9.32 For the circuit given in Figure 15, assume that the switch has been open for a long time before closing at $t = 0$. Find the voltage $v_x(t)$ for $t > 0$.

9.33 For the circuit given in Figure 16, determine all labeled currents and voltages immediately after the switch is closed and after a very long time. Assume that the switch has been open for a very long time before closing. Is this circuit a series or parallel *RLC* circuit?

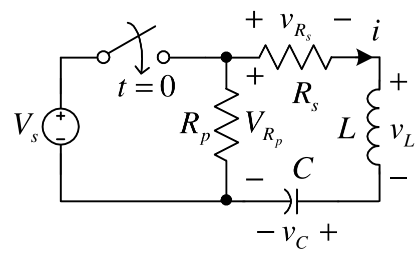

Figure 16

9.34 For the circuit given in Figure 17, determine all labeled currents and voltages immediately after the switch is opened and after a very long time. Assume that the switch has been closed for a very long time before opening. Is this circuit a series or parallel *RLC* circuit?

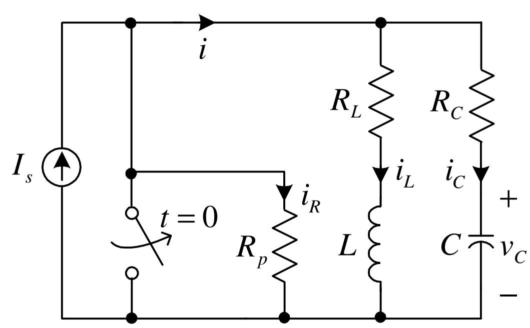

Figure 17

9.35 For the circuit given in Figure 18, determine all labeled currents and voltages immediately after both switches are closed and after a very long time. Assume that the switches have been open for a very long time before closing. Is this circuit a series or parallel *RLC* circuit?

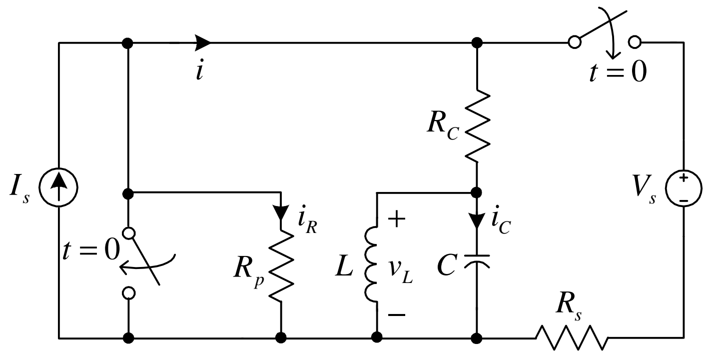

Figure 18

9.36 Determine and sketch $i(t)$ and $v_{100}(t)$ for the circuit given in Figure 19 after the switch is closed. Is this system oscillatory? Then, determine and sketch the energy dissipated in the resistor and energy stored in the inductor and capacitor after the switch is closed. Approximately, how long does it take the transients to "die off."

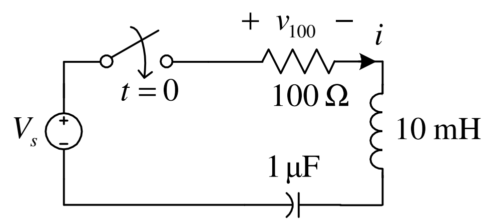

Figure 19

9.37 For the circuit given in Figure 20, if $I_s = 10$ mA, $R = 10\ \Omega$, $L = 2.2\ \mu$H, and $C = 0.01\ \mu$F, determine and sketch $i(t)$, $i_R(t)$, $i_L(t)$, and $v(t)$ after the switch is thrown. Is this system oscillatory? Determine and sketch the energy dissipated by the resistor and energy stored by inductor and capacitor for $t > 0$. Approximately, how long does it take the transients to "die off."

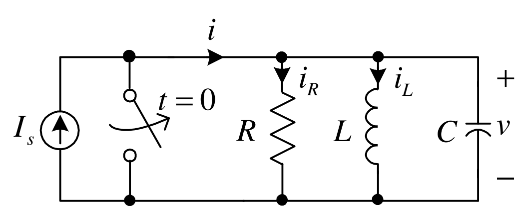

Figure 20

9.38C Determine the expression for $i(t)$, $v_{Rx}(t)$, and $v_C(t)$ for the circuit given in Figure 21 after the switch is thrown open. Assume that $V_s = 9$ V, $R_s = 10\ \Omega$, $L = 10$ mH, and $C = 0.001\ \mu$F. Select a value for R_x so that the system is oscillatory with a ringing frequency of 30 kHz. Then, select another value for R_x so that the system is overdamped with a negligible response after 10 ms.

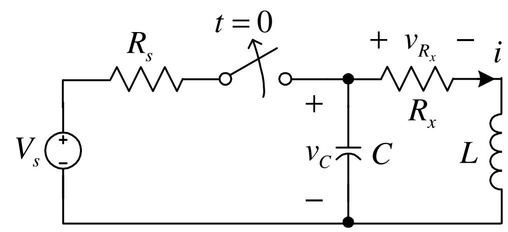

Figure 21

9.39EC Assuming that the switch has been open for a long time before closing at $t = 0$, determine the expressions for $v_C(t)$ and $i_x(t)$ for the circuit given in Figure 22. Use superposition to determine the initial values. Assume that $V_s = 6$ V, $I_s = 100$ mA, $R_s = 10\ \Omega$, $R_x = 220\ \Omega$, $R_p = 10$ kΩ, $L = 1$ mH, and $C = 0.22$ μF. Estimate the time required for the transients to decay to a negligible level. Is this a series or parallel *RLC* circuit?

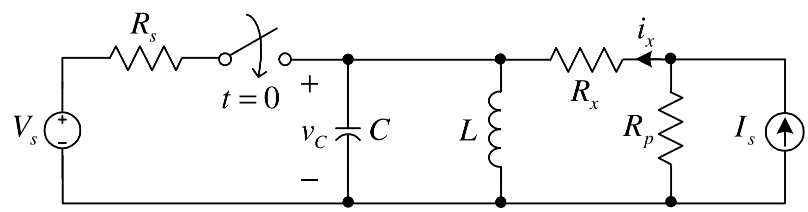

Figure 22

9.40EC Assuming that the switch has been closed for a long time before opening at $t = 0$ for the circuit given in Figure 23, determine the expressions for $v_L(t)$ and $i_x(t)$. Use superposition to determine the initial values. Assume that $I_x = 10$ mA, $I_s = 20$ mA, $R_x = 22$ kΩ, $R_y = 10$ kΩ, $L = 100$ mH, and $C = 2.2$ μF. Estimate the time required for the transients to decay to a negligible level. What is smallest possible ringing frequency if R_x is allowed to vary? Is this a series or parallel *RLC* circuit?

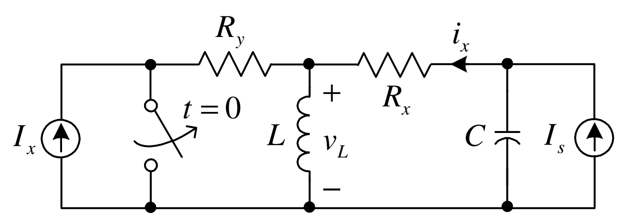

Figure 23

9.41C Assuming that the switch has been open for a long time before closing at $t = t_x$ for the circuit given in Figure 24, determine the expressions for $v_C(t)$ and $i_x(t)$.

Assume that $V_s = 15$ V, $R_x = 2.2$ kΩ, $R_y = 4.7$ kΩ, $R_z = 6.8$ kΩ, $L = 10$ mH, and $C = 3.3$ μF. Estimate the time required for the transients to decay to a negligible level. Is this a series or parallel *RLC* circuit?

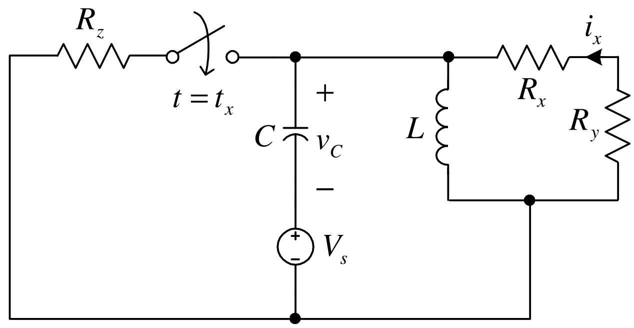

Figure 24

9.42 Using real component values, design a circuit capable of oscillating for 1 minute with only a 1% reduction in amplitude at a frequency of 10 kHz. The amplitude of the initial oscillation should be 10 mV. Repeat the design for a 10 MHz ringing frequency. Which specification is easier to meet and why?

9.43E Derive the equations for the damping coefficient and resonant frequency given in the chapter for the network consisting of two series-*RL* circuits that are in parallel. Check either the damping coefficient or resonant frequency against another known result by allowing certain terms to approach zero or infinity.

9.44E Derive the equations for the damping coefficient and the resonant frequency given in the chapter for the network consisting of two series-*RC* circuits that are in parallel Check either the damping coefficient or resonant frequency against another known result by allowing certain terms to approach zero or infinity.

9.45E Derive the equations for the damping coefficient and resonant frequency given in the chapter for the network consisting of a parallel *RL* circuit in series with a parallel *RC* circuit. Check either the damping coefficient or resonant frequency against another known result by allowing certain terms to approach zero or infinity.

9.46E Derive the equations for the damping coefficient and resonant frequency given in this chapter for the network consisting of a parallel *RL* circuit in series with another parallel *RL* circuit. Check either the damping coefficient or resonant frequency against another known result by allowing certain terms to approach zero or infinity.

9.47E. Derive the equations for the damping coefficient and resonant frequency given in the chapter for the network consisting of a parallel *RC* circuit in series with another parallel *RC* circuit. Check either the damping coefficient or resonant frequency against another known result by allowing certain terms to approach zero or infinity.

9.48E Determine the expression for the voltage across R_1 for the circuit given in Figure 25, assuming that the switch has been opened for a long time before closing at *t*

= 0. Clearly show the polarity of this voltage. Let $V_s = 6$ V, $R = 22\ \Omega$, $R_1 = 2\ \Omega$, $R_2 = 4\ \Omega$, $L_1 = 2$ mH, and $L_2 = 10$ mH.

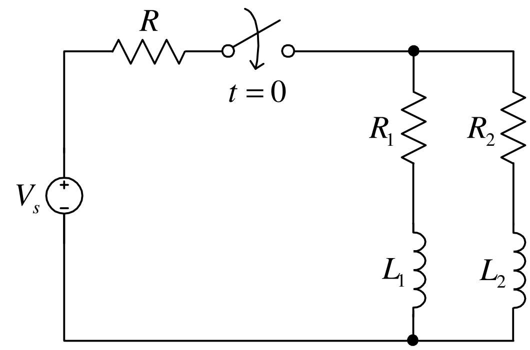

Figure 25

9.49E Determine the expression for the current through C_1 for the circuit given in Figure 26, assuming that the switch has been opened for a long time before closing at $t = 0$. Clearly show the direction of this current. Let $V_s = 9$ V, $R = 2\ \Omega$, $R_1 = 220\ \Omega$, $R_2 = 2$ kΩ, $C_1 = 0.22\ \mu$F, and $C_2 = 1.0\ \mu$F.

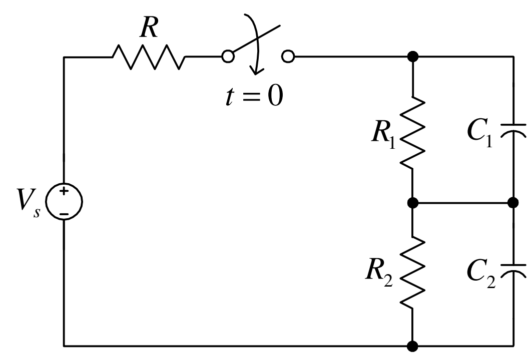

Figure 26

9.50E Determine the expression for the voltage across L for the circuit given in Figure 27, assuming that the switch has been opened for a long time before closing at $t = 0$. Clearly show the polarity of this voltage. Let $V_s = 15$ V, $R = 22\ \Omega$, $R_1 = 1$ kΩ, $R_2 = 22$ kΩ, $C = 2.2\ \mu$F, and $L = 100$ mH.

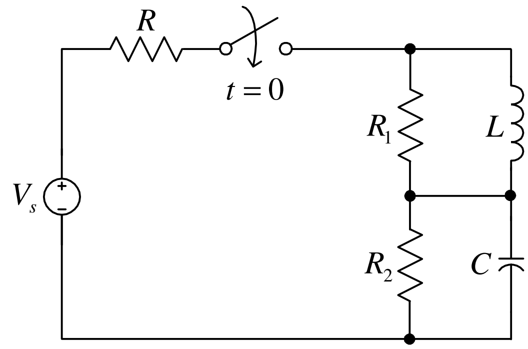

Figure 27

9.51EC Determine whether the given circuit in Figure 28 oscillates. If it does oscillate, determine its frequency and the number of whole integer cycles that occur before the oscillatory signal falls below *approximately* 25% of its maximum amplitude. For all of the switching intervals, estimate the time required for the respective transients (voltage for capacitors and current for inductors) to decay within 5% of their final value. Finally, estimate the current through each inductor and voltage across each capacitor at $t = t_x$, and determine their exact values at $t = \infty$. Let $V_s = 12$ V, $R = 22$ Ω, $R_1 = 1$ kΩ, $R_2 = 22$ kΩ, $C_1 = 2.2$ μF, $C_2 = 0.22$ μC, and $t_x = 2$ ms.

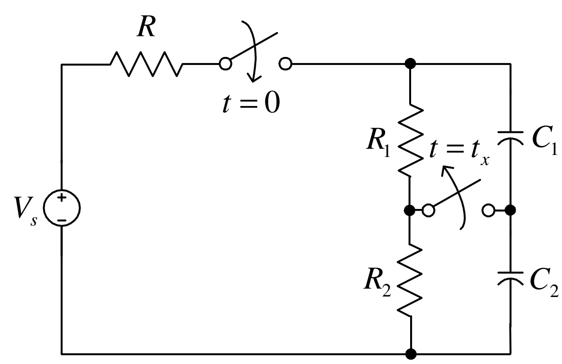

Figure 28

9.52EC Repeat Problem 9.51 for the circuit given in Figure 29. Let $V_s = 6$ V, $R = 220$ Ω, $R_1 = 10$ Ω, $R_2 = 10$ kΩ, $R_3 = 10$ Ω, $C = 22$ μF, $L = 100$ mH, and $t_x = 10$ ms.

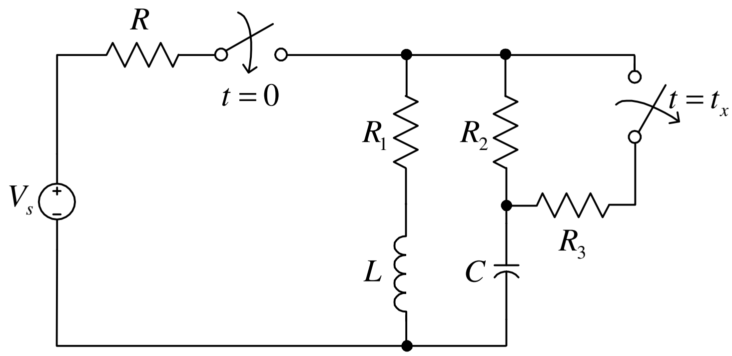

Figure 29

9.53EC Repeat Problem 9.51 for circuit given in Figure 30. Let V_s = 12 V, R = 1 Ω, R_1 = 10 kΩ, R_2 = 22 kΩ, C = 0.022 μF, L = 10 mH, and t_x = 200 μs.

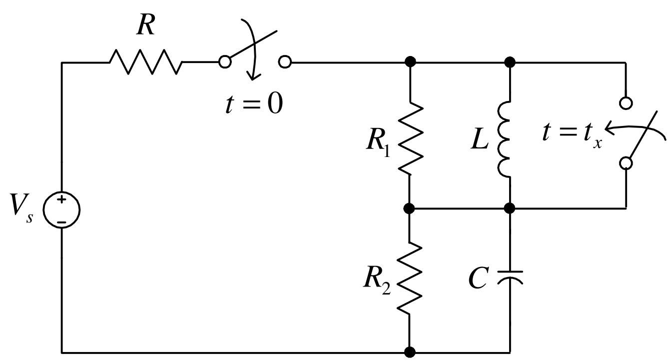

Figure 30

9.54EC Repeat Problem 9.51 for circuit given in Figure 31. Let I_s = 10 mA, R = 220 Ω, R_1 = 10 Ω, R_2 = 10 kΩ, R_3 = 10 kΩ, L_1 = 220 mH, L_2 = 10 mH, and t_x = 2 ms. Hint: source transformations.

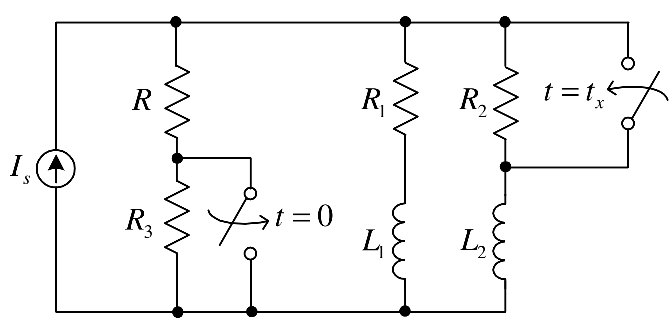

Figure 31

9.55 Verify that for the function

$$x(t) = A\frac{\omega_o}{\omega_d}e^{-\alpha t}\sin\left[\omega_d t + \tan^{-1}\left(\frac{\omega_d}{\alpha}\right)\right] + F$$

the maximum percent overshoot is

$$\%\text{ overshoot } = 100\frac{A}{F}e^{-\frac{\pi\alpha}{\sqrt{\omega_o^2 - \alpha^2}}}$$