Chapter 7: Passive Filters

7.1 Repeat the analytical analysis given in this chapter for the low-pass *RC* filter for an *RC* filter in shunt with the load. The *R* and *C* for this filter are in series.

7.2 Inductors are often used in series with lines to block noise currents in low-impedance circuits. However, if the cable is carrying a fast digital signal, what is a possible negative consequence of using this type of filter?

7.3C An *LC* filter is to be used to filter out high-frequency noise. The source impedance is low (0.1 Ω), and the load impedance is low (10 Ω). Using a numerical package, design a filter to filter 10 MHz noise. The filter response should be at least 10 dB down at 10 MHz relative to 100 kHz. Resonance "peaks" should not exist in the filter response from 1 kHz to 100 MHz. The insertion loss of the filter at 100 kHz should be less than 1 dB. Plot the magnitude of the response in dB from 1 kHz to 100 MHz. The *LC* filter should use standard nonideal components. If these filter specifications cannot be met, state specifically why. Repeat the entire analysis for a "*CL*" filter. Which filter is more appropriate for these impedances?

7.4C Repeat Problem 7.3 for a source impedance that is high (1 kΩ) and a load impedance that is low (10 Ω). If these filter specifications cannot be met, state specifically why.

7.5C Repeat Problem 7.3 for a source impedance that is high (1 kΩ) and a load impedance that is high (100 kΩ). If these filter specifications cannot be met, state specifically why.

7.6 Show that for the band-reject transfer function

$$\frac{V_L}{V_s} = \frac{-\omega^2 + \omega_c^2}{-\omega^2 + j\omega\left(BW\right) + \omega_c^2}$$

the bandwidth and frequency response are *BW* and ω_c, respectively.

7.7C A π filter (two shunt capacitors with a series inductor between them) is to be used to filter out high-frequency noise. The source impedance is low (0.1 Ω), and the load impedance is low (10 Ω). Using a numerical package, design a filter to filter 10 MHz noise. The filter response should be at least 10 dB down at 10 MHz relative to 100 kHz. Resonance "peaks" should not exist in the filter response from 1 kHz to 100 MHz. The insertion loss of the filter at 100 kHz should be less than 1 dB. Plot the magnitude of the response in dB from 1 kHz to 100 MHz. The π filter should use standard nonideal components. If these filter specifications cannot be met, state specifically why. Repeat the entire analysis for a T filter (two series inductors with shunt capacitor between them). Which filter is more appropriate for these impedances?

7.8C Repeat Problem 7.7 for a source impedance that is low (0.1 Ω) and a load impedance that is high (100 kΩ). If these filter specifications cannot be met, state specifically why.

7.9C Repeat Problem 7.7 for a source impedance that is high (1 kΩ) and a load impedance that is low (10 Ω). If these filter specifications cannot be met, state specifically why.

7.10C A band-pass filter (parallel *LC* in shunt with the load) is to be used to pass a desired signal but reject nearby noise. The source impedance is low (0.1 Ω), and the load impedance is low (10 Ω). Using a numerical package, design a filter to pass a 10 MHz signal. The filter response should be at least 10 dB down at 1 MHz and 100 MHz relative to 10 MHz. Resonance "peaks" should not exist in the filter response for frequencies less than 1 MHz or greater than 100 MHz. Resonance "dips" should not exist in the filter response from 1 MHz to 100 MHz. The insertion loss of the filter at 10 MHz should be less than 1 dB. Plot the magnitude of the response in dB from 1 kHz to 500 MHz. The band-pass filter should use standard nonideal components. If these filter specifications cannot be met, state specifically why.

7.11C A band-reject filter (series *LC* in shunt with the load) is to be used to reject noise but pass nearby desirable signals. The source impedance is low (0.1 Ω), and the load impedance is low (10 Ω). Using a numerical package, design a filter to reject a 10 MHz noise signal. The filter response at 10 MHz should be at least 10 dB down relative to 1 MHz and 100 MHz. Resonance "dips" should not exist in the filter response for frequencies less than 1 MHz or greater 100 MHz (but less than 300 MHz). Resonance "peaks" should not exist in the filter response between 1 MHz and 100 MHz (but less than 300 MHz). The insertion loss of the filter at 1 MHz and 100 MHz should be less than 1 dB. Plot the magnitude of the response in dB from 1 kHz to 500 MHz. The band-reject filter should use standard nonideal components. If these filter specifications cannot be met, state specifically why.

7.12 Determine the transfer function (load voltage divided by the source voltage) for the filter given in Figure 1. Sketch its Bode magnitude plot including break frequencies, slopes, and amplitudes. Then, sketch the Bode magnitude plot of the filter's input impedance. (Both of these plots should only be a function of the given component variables.) Determine any advantages and disadvantages of this filter compared to the same filter without *R*.

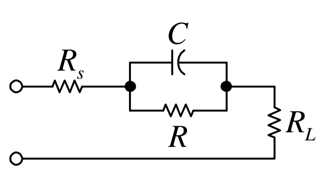

Figure 1

7.13 Repeat Problem 7.12 for the filter given in Figure 2.

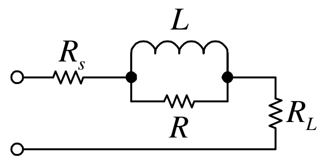

Figure 2

7.14C Design a low-pass video amplifier filter with a cutoff frequency of about 320 kHz. The attenuation beyond this frequency should be about 60 dB/decade. Assuming that the load is 1 kΩ, plot the input impedance (magnitude and phase angle) of this filter (not including the source resistance) and the transfer voltage (magnitude and phase angle) of the filter (including the source resistance) from 10 kHz to 10 MHz when the source resistance is equal to 10 Ω, 100 Ω, 1 kΩ, 10 kΩ, and 100 kΩ. Which of these source resistances provides (over the passband) the flattest voltage response, most linear voltage phase response, the greatest voltage overshoot (in the frequency domain), the greatest voltage ripple (in the frequency domain), and the fastest voltage transition at cutoff?

7.15EC Determine the transfer function (load voltage divided by the source voltage) for the L-type filter shown in Figure 3. Sketch its Bode magnitude plot, ignoring any overshoot terms. Are the low frequencies "boosted" and the high frequencies attenuated? It is stated that the input impedance is relatively constant with R_L variation while the impedance seen by the load is quite variable with R_s variation. Determine whether this statement is possibly true by numerically analyzing the equation for the input and output impedance. (For the output impedance, assume that the source resistance is equal to R_s.)

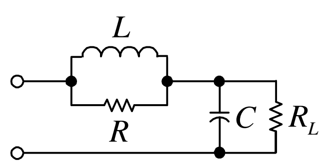

Figure 3

7.16EC Determine the transfer function (load voltage divided by the source voltage) for the L-type filter shown in Figure 4. Sketch its Bode magnitude plot, ignoring any overshoot terms. Are the high frequencies "boosted" and the low frequencies attenuated? It is stated that the input impedance is relatively constant with R_L variation while the impedance seen by the load is quite variable with R_s variation. Determine whether this statement is possibly true by numerically analyzing the equation for the input and output impedance. (For the output impedance, assume that the source resistance is equal to R_s.)

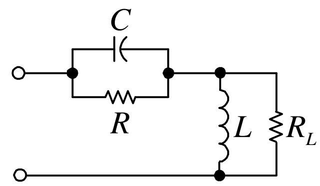

Figure 4

7.17C It is commonly believed that the response of π filters are less sensitive to the source and load impedance when the source and load impedance are large. Numerically determine if and when this is true.

7.18C It is commonly believed that the response of T filters are less sensitive to the source and load impedance when the source and load impedance are small. Numerically determine if and when this is true.

7.19 Verify the Bode magnitude plots for the voltage transfer function and the input impedance for the multiple *C*, *RC* filter # (provided by your instructor) provided in this chapter. Since there are no inductors present in these circuits, there is no overshoot. Therefore, do not neglect any "overshoot-like" terms. Check all break frequencies, slopes, and amplitudes.

7.20C Determine the Bode magnitude plots for the voltage transfer function and for the input impedance for the *RC* filters given in Figure 5, Figure 6, and Figure 7 . Since there are no inductors present in these circuits, there is no overshoot. Therefore, do not neglect any "overshoot-like" terms. Label all break frequencies, slopes, and amplitudes.

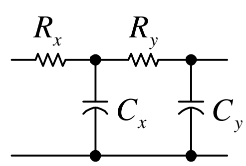

Figure 5

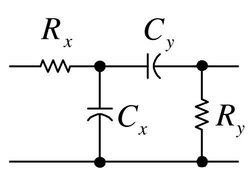

Figure 6

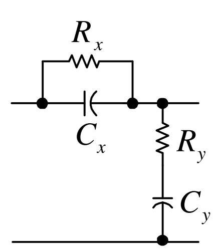

Figure 7

7.21 Derive the equations provided in this chapter in the high-Q discussion for the series-to-parallel transformation of an RL circuit. Work with admittances instead of impedances.

7.22 Derive the results provided in this chapter in the high-Q discussion for the series-to-parallel transformation of an RC circuit.

7.23 Derive the results provided in this chapter in the high-Q discussion for the parallel-to-series transformation of an RC circuit.

7.24C Plot the error versus Q associated with using the high-Q approximations for resistors, inductors, and capacitors for both the series-to-parallel and parallel-to-series transformations. Allow the Q to range from 1 to 100.

7.25 As stated in this chapter, the Q of a network is defined as 2π times the ratio of the maximum energy stored to the energy lost each period of the excitation frequency. Starting with this energy definition, verify that the Q of circuit # (provided by your instructor) in the Q table in this chapter is the ratio of the reactance to the resistance of the impedance of the circuit.

7.26 It is stated in another book that increasing the resistance of a linear, passive RLC circuit will reduce its Q. Find two counterexamples to show that this statement is not necessarily true.

7.27 If both the inductor and capacitor in the circuit shown in Figure 8 are modeled using a different resistor in shunt with each of these passive elements (as with the cases discussed in this chapter), determine the percent error between the desired versus actual overall Q of the circuit if the Q of the L and C are twenty times the desired Q. Do not assume that R is small.

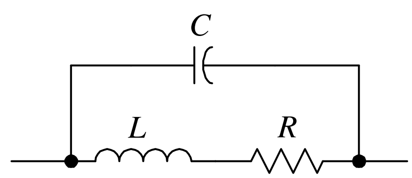

Figure 8

7.28 Repeat Problem 7.27 but assume that both the inductor and capacitor are modeled using a different resistor in series with the L and the C.

7.29 The Q of an inductor is determined by measuring the response of the inductor (frequency corresponding to the maximum response and frequencies corresponding to the 3 dB points) when placed in parallel with a capacitor. The Q of the inductor measured in this manner is actually less than the actual Q of the inductor. Using an appropriate model for a real inductor, explain why this is true.

7.30 One simple model for a crystal is shown in Figure 9.

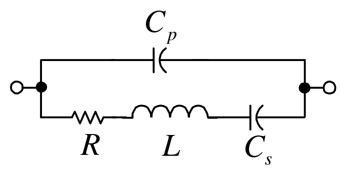

Figure 9

What parameter is not included in this model? Given that the series and parallel resonant frequencies are, respectively,

$$\omega_s = \frac{1}{\sqrt{LC_s}}, \quad \omega_p = \omega_s\sqrt{1+\frac{C_s}{C_p}}$$

show that if the "distance" between these two resonances is $\Delta\omega = \omega_p - \omega_s$, then

$$\frac{C_s}{C_p} = \frac{\omega_p^2 - \omega_s^2}{\omega_s^2} \approx \frac{2\Delta\omega}{\omega_s}$$

If the load across the crystal is mainly capacitive in nature, how is the resonant frequency affected by this load?

7.31 By assuming that $Q_L = \omega L/R > 5$, determine approximate expressions for the total Q of the circuit given in Figure 10 at both $\omega = 1/\sqrt{LC}$ and at the true resonant frequency of the circuit.

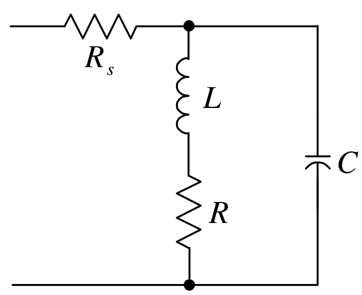

Figure 10

7.32C Design a band-pass filter with a center frequency of 10 kHz and a bandwidth of about 800 Hz assuming that the source resistance is 10 Ω and the load resistance is 10 kΩ. The insertion loss of the filter at 10 kHz should be less than 3 dB. Plot the magnitude and phase of the voltage transfer function from 9 to 11 kHz. Is the magnitude response flat over the given bandwidth? Is the phase response linear over the given bandwidth? Then, plot the input and output voltages versus time (on the same plot) for both of the following input signals:

$$x(t) = 2\cos\left[2\pi\left(10\times10^3 - 400\right)t + 20^\circ\right] - 0.5\cos\left[2\pi\left(10\times10^3\right)t + 70^\circ\right]$$
$$x(t) = 2\cos\left[2\pi\left(10\times10^3 - 1000\right)t + 20^\circ\right] - 0.5\cos\left[2\pi\left(10\times10^3\right)t + 70^\circ\right]$$

Explain why the output is or is not distorted in both cases.

7.33C Assume a crystal is modeled by the following parameters:

$$L = 0.058\ \text{H},\ C_s = 0.018\ \text{pF},\ R = 8\ \Omega,\ C_p = 4\ \text{pF}$$

Numerically verify all of the consequences given in this chapter when a resistor, a capacitor, and an inductor are added in series and shunt with the crystal. The magnitude of the total impedance of the crystal with the external element should be plotted versus frequency for each case.

7.34 The relationship between the phase and the frequency of a filter is given by

$$\theta = \omega t_d + \phi$$

When will phase distortion occur? Explain.

7.35C Plot the magnitude response in dB and the phase response of the multipole filter given in Figure 11. The frequency range of the plot should be at least an order of magnitude above and below the "major" cutoff or center frequency of the filter. Based on the filter descriptions provided in this chapter and these plots, name this filter type (e.g., low-pass Bessel). Provide your reasoning.

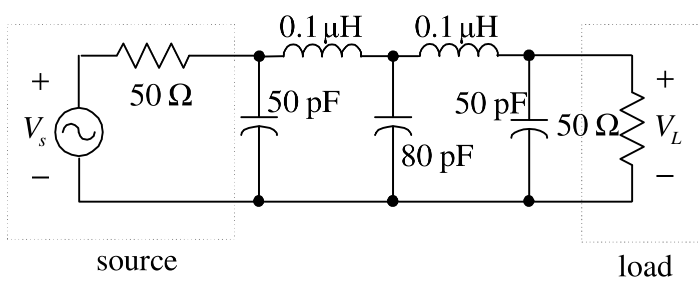

Figure 11

7.36C Repeat Problem 7.35 for the following transfer function:

$$H(s) = \frac{1}{s^4 + 2.61s^3 + 3.41s^2 + 2.61s + 1} \quad \text{where } s = j\omega$$

7.37 Determine the equation for the insertion loss of an *LC* filter. Simplify the expression.

7.38 Determine the equation for the insertion loss of a *CL* filter. Simplify the expression.

7.39C It is stated that for multisection filters, the load and source have less effect on the *shape* of the overall response as the number of sections increase. Determine whether this statement is reasonable by plotting and comparing the magnitude of the frequency response for a one-section, two-section, and three-section *LC* filter. For the one section filter let $L = 1$ mH and $C = 1$ μF. For the two section filter, let each section have the values $L = 1/2$ mH and $C = 1/2$ μF. For the three section filter, let each section have the values $L = 1/3$ mH and $C = 1/3$ μF. For each filter, plot the response for all nine possible combinations of 1 Ω, 30 Ω, and 10 kΩ resistances for the source and load.

7.40 Physically explain why a series *RLC* circuit is capacitive for frequencies less than its resonant frequency and inductive for frequencies greater than its resonant frequency. Then, physically explain why a parallel *RLC* circuit is inductive for frequencies less than its resonant frequency and capacitive for frequencies greater than its resonant frequency.

7.41 Show that *at resonance* the *LC* portion of a series *RLC* circuit appears like a short circuit and *LC* portion of a parallel *RLC* circuit appears like an open circuit.

7.42C Plot, on the same set of axes, the total instantaneous energy in a series *RLC* circuit versus time (over one complete cycle) if $R = 30$ Ω, $L = 1$ mH, and $C =$ 0.01 μF for $\omega = \omega_o/10$, $\omega = \omega_o$, and $\omega = 10\omega_o$. Repeat this numerical analysis if $R = 3$ Ω and $R = 300$ Ω.

7.43C Numerically show when $R = 300$ Ω, $L = 1$ mH, and $C = 0.01$ μF that the maximum value of the amplitude responses of the resistor voltage, capacitor voltage, and inductor voltage for a series *RLC* circuit are not the same and are equal to the values given by the equations given in this chapter.

7.44EC Show that the bandwidth of the capacitor's voltage amplitude response for a series *RLC* circuit is approximately equal to the bandwidth of the circuit's current response if the Q is high. First, assume that the maximum response occurs at resonance for the capacitor's voltage amplitude. Second, determine the amplitude of this response at resonance. Third, divide this maximum response at resonance by $\sqrt{2}$ and set it equal to the amplitude response, which is a function of the frequency. Fourth, solve for the frequencies corresponding to the 3 dB points. Fifth, subtract the real positive upper 3 dB frequency by the real positive lower 3 dB frequency to determine an expression for the bandwidth. Sixth, rewrite most of this bandwidth expression in terms of the Q at resonance:

$$Q_o = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Finally, assume that the Q at resonance is large and use the following approximation valid when x is small:

$$\sqrt{1+x} \approx 1 + \frac{1}{2}x$$

7.45EC Repeat the analysis given in Problem 7.44 for the inductor's voltage amplitude response for a series *RLC* circuit.

7.46C Repeat the analysis given in this chapter for the 3-terminal capacitor but include a reasonable resistive term for each of the beaded leads. How does the performance of the capacitor change?

7.47C Repeat the analysis given in this chapter for the 3-terminal capacitor but use a balanced three-terminal capacitor (two leads from both sides of the capacitor).

7.48C Determine the bandwidth of a short-circuited one-quarter wavelength 75 Ω coaxial filter with a center frequency of 144 MHz.

7.49C Determine the bandwidth of an open-circuited one-quarter wavelength 75 Ω coaxial filter with a center frequency of 144 MHz.