# Chapter 30: Antennas

30.1 Can a lumped-circuit model be used to represent an electrically-large dipole? If yes, indicate when. If no, why is it represented by a radiation resistance and reactance?

30.2 Above what frequency should twin-lead transmission lines not normally be used? State all assumptions.

30.3 Derive the equation given in this chapter for the ohmic resistance of an electrically-short dipole of length $2l_{th}$ if the current distribution is uniform along its length:

$$I(z) = \begin{cases} I_o & |z| < l_{th} \\ 0 & |z| > l_{th} \end{cases}$$

Assume that the ac resistance per unit length of the conductors is $r_{AC}$.

30.4 Derive the equation given in this chapter for the ohmic resistance of a dipole of length $2l_{th}$ if the current distribution is described by

$$I(z) = \begin{cases} I_o\left[\frac{(k-1)}{a}|z|+1\right] & |z| < a \\ \frac{kI_o}{l_{th}-a}\left(l_{th}-|z|\right) & a < |z| < l_{th} \\ 0 & |z| > l_{th} \end{cases}$$

Assume that the ac resistance per unit length of the conductors is $r_{AC}$.

30.5 Verify that the total ohmic resistance is $l_{th}r_{AC}$ rather than $2l_{th}r_{AC}$ for a large dipole with a sinusoidal current distribution

$$I(z) = I_o \sin\left[\frac{2\pi}{\lambda}\left(l_{th}-|z|\right)\right]$$

where $2l_{th} = \lambda/2$. Assume that the ac resistance per unit length of the conductors is $r_{AC}$.

30.6 A monopole antenna used for a remote-controlled device operating at about 75 MHz was designed to be about 2 ft in length. Unfortunately, because of size requirements, the antenna must be reduced to 4 inches. (Assume the antenna is constructed of #22 AWG copper wire.) Determine the input impedance of the monopole antenna for both the 2 ft and 4 inch lengths. If the difference in these impedances is too dramatic for the transmitter, connected to the antenna, propose a method of reducing this impedance change while still retaining the smaller size requirements.

30.7 A half-wave dipole operating at a free-space wavelength of 160 m is constructed of #10 AWG annealed copper wire. Is the skin depth important? For a sinusoidal current distribution on the antenna, determine the total ohmic power loss for the antenna if the maximum current amplitude along the wire is 5 A. One quarter of each side of the wire near the feed line is then coated with silver. Determine the new total power loss for the antenna. (Select a reasonable silver coating thickness.) Why is the silver applied near the feed line instead of at the ends of the antenna?

30.8 It is stated that the directivity and input impedance of a (5/8)$\lambda$ vertical monopole is greater than a $\lambda$/4 vertical monopole. Determine whether this is true. Then, compare the location of the current maximum for a these two antenna. Determine the total ohmic resistance for each of these antennas assuming the ac resistance per unit length is equal to $r_{AC}$. If the ground is acting as the image for the monopole antenna (i.e., the other side of the dipole), which antenna will have the greater ground losses? Determine and compare the radiation resistances of the two antennas. Which antenna is more efficient? Assume that the current distribution on the antenna is given by

$$I(z) = I_o \sin\left[\frac{2\pi}{\lambda}\left(l_{th} - |z|\right)\right]$$

Often radial wires are used to increase the effectiveness of the ground plane for real antenna. The length of these radials should be greater for the (5/8)$\lambda$ vertical antenna than the $\lambda$/4 monopole. Why does this seem reasonable?

30.9ES For a piece wire of specific length that is electrically short, what wire shape will provide the greatest radiation resistance?

30.10C The radiation resistance of a half-wave dipole antenna constructed of #16 AWG copper wire is 73 + $j$43 Ω. A 100 ft 75 Ω coax connects a 21 MHz transmitter (with a source impedance of 50 Ω) with an open-circuit source voltage of 40 V to this dipole antenna. Determine the power radiated by the antenna. Determine the efficiency of the antenna.

30.11 The radiation resistance of top-loaded short dipole is greater than a nontop-loaded short dipole (of the same length). Explain why this is true. Placing an inductor in series with the antenna can also increase the radiation resistance. Explain why this is true. Placing the inductor midway along the antenna is more efficient than near the source. Explain why this is true. [Collin, '85; Watt]

30.12 Why is the capacitance of an electrically-short loop small?

30.13C Compare the reactance of a $\lambda$/10 length straight-wire antenna using the short antenna approximation and the large antenna equation. Assume three different (but reasonable) radii for the wire.

30.14 Why is the radiation efficiency difficult to measure for short antenna? Propose a method of measuring the efficiency.

30.15 It is stated that the radiation resistance is much less than the ohmic resistance for electrically-short loop antenna. Under what conditions is this statement true?

30.16C For large dipole antenna, plot the radiation resistance versus $a/\lambda$ at resonance (where $a$ is the radius of the conductor). Why is this information useful?

30.17C For large dipole antenna, plot $a/\lambda$ versus $2l_{th}/\lambda$, where $l_{th}$ is the length corresponding to resonance and $a$ is the radius of the conductor. Why is this information useful?

30.18 One method to fine-tune a whip antenna is to measure the VSWR before and after touching a metallic object (appropriately insulated and separated from the individual performing the test) to the end of the antenna. If the VSWR increases, the antenna is too long. If the VSWR decreases, the antenna is too short. Explain why this relationship is valid. Would the size of the metallic object, for a comparable change in VSWR, be smaller or larger as the frequency of operation decreases?

30.19C Does the half-wave dipole have the greatest electric field (magnitude) if the total length is limited to less than or equal to $\lambda$? Plot the field versus the angle ($\theta$) for total dipole lengths varying from $\lambda/10$ to $2\lambda$ in $\lambda/10$ increments.

30.20 A strip-line. susceptibility test system consists of a narrow strip above a large ground plane that is fed at one end with a broadband transmitter and resistive-impedance matched at the other end. The line is 1.5 m in length, and the distance between the strip and the ground plane is 0.2 m. The system is used from 10 kHz to 200 MHz to couple strong fields into harnesses placed between the strip line and ground plane. The power loss, the ratio of the load power to input power, is nearly 20 dB at 200 MHz. If the ohmic resistance is relatively small, why is the power loss so large at the higher frequencies?

30.21 In the far field in the cylindrical coordinated system, determine the equation for the directive gain. State all assumptions.

30.22C Plot the VSWR of a half-wave dipole (constructed of #18 AWG copper wire) that is resonant at 28 MHz as the frequency varies from 26 to 30 MHz. Determine the bandwidth and $Q$ of this antenna. Assume that the impedance of the transmission line connected to the antenna is 75 Ω.

30.23C Determine the bandwidth of a 27 MHz half-wave dipole based on its radiation pattern variation. State all assumptions.

30.24 Show that the VSWR can be written in terms of the $Q$ of an antenna:

$$\mathrm{VSWR} = \frac{1 + \dfrac{Qn}{\sqrt{(Qn)^2 + 1}}}{1 - \dfrac{Qn}{\sqrt{(Qn)^2 + 1}}}$$

where $n$ is the percent difference, expressed as a decimal (e.g., 0.05 corresponding to 5%) between the antenna's resonant frequency and frequency where the $Q$ is measured. Assume that the antenna is matched to the transmission line at its center resonant frequency. Plot the VSWR versus $Q$ (ranging from 0.1 to 10) for $n$ = 0.01, 0.02, 0.03, 0.04, and 0.05. Use a logarithmic scale for the $Q$ axis.

30.25 Determine the bandwidth and $Q$ of an electrically-small loop antenna with a center frequency of 144 MHz. Assume that the loop is constructed of copper of a “reasonable” gauge.

30.26C Show that the directivity of a half-wave dipole is 1.64 beginning with the basic definition provided in this chapter:

$$G_d = \frac{P_d}{P_{avg}}$$

30.27C Show that the directivity of a full-wave dipole is 2.41 beginning with the basic definition provided in this chapter:

$$G_d = \frac{P_d}{P_{avg}}$$

30.28C Numerically verify that for a dipole, sidelobes (submaximum lobes) begin to appear for lengths greater than one wavelength.

30.29 Beginning with the basic definition for directivity, determine the directivity of an antenna with the given far field:

$$E_{\phi s} = -\eta_o H_{\theta s},\ H_{\theta s} = \left(\frac{ka}{2}\right)^2 I_o \frac{e^{-jkr}}{r} \sin\theta \quad \text{where } k = \frac{2\pi}{\beta}$$

30.30 Consider the four-port, linear, time-invariant, zero-state, passive network given in Figure 1. With the two different current sources connected to two of the ports, the open-circuit voltages across the remaining two ports are as shown.

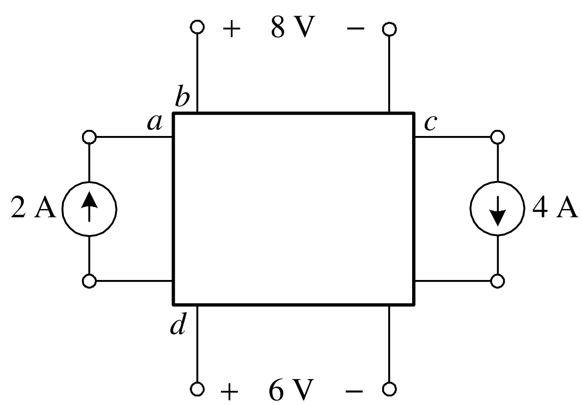


**Figure 1**

Now, consider the new excitations shown in Figure 2. Only one open-circuit voltage is measured. Using the Law of Reciprocity, determine the unknown open-circuit voltage $V_x$.

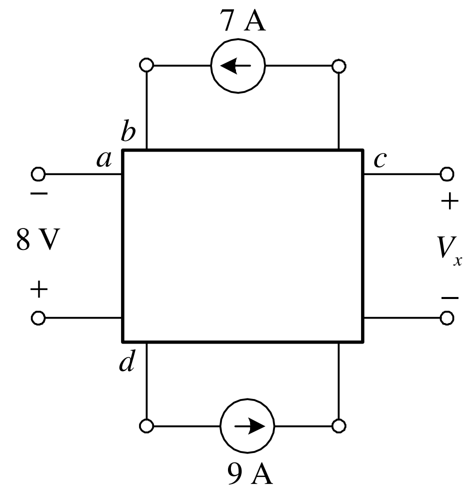


**Figure 2**

30.31 Hoping to modify easily the radiation pattern of a single dipole antenna, a student places a variable *LC* network at the feedpoint of the antenna. Will adjusting the *L* and *C* values change the radiation pattern? Explain.

30.32 A set of antennas is to transmit and receive over the frequency range 30 kHz to 300 MHz. Specify a set of antennas that will meet this criteria.

30.33 Is a biconical antenna balanced or unbalanced? Explain.

30.34S Provide one common non-EMC use for each of the broadband antennas listed.

30.35S It is stated that adding two ridges to a standard horn antenna increases its bandwidth by reducing the cutoff frequency of the dominant mode while increasing the cutoff frequency of the next higher mode. Determine the validity of this statement.

30.36 As the gain of an antenna increases, the main beam's width decreases (the classical gain-bandwidth product). Because of this, the distance between the antenna and the test source, *r*, for EMC measurements should follow the guideline

$$r \geq \frac{D\lambda}{\pi}$$

to avoid excessive coupling, where *D* is the directivity of the antenna. Derive this equation and determine its limitations. [Bronaugh; Johnson, '61]]

30.37 Determine the direction and the polarization (linear, circular, or elliptical) of each of the following waves:

$$\vec{E} = 3\cos(\omega t - \beta z)\hat{a}_x + 3\sin(\omega t - \beta z)\hat{a}_y \text{ mV/m}$$
$$\vec{E} = 3\cos(\omega t - \beta y)\hat{a}_z \text{ μV/m}$$
$$\vec{E} = 3\cos(\omega t - \beta y)\hat{a}_x + 2\sin(\omega t - \beta y)\hat{a}_z \text{ mV/m}$$
$$\vec{E} = 2\cos(\omega t - \beta x + 35^\circ)\hat{a}_z + 3\cos(\omega t - \beta x)\hat{a}_y \text{ μV/m}$$

Sketch the fields versus time.

30.38 It is stated that for a quarter-wave long transmission line, the magnitude of the current in the load is equal to the voltage at the source (voltage at the input of the line) divided by the characteristic impedance of the line; that is, the load current (current at the output of the line) is independent of the load impedance! Determine when this is true. Why is a quarter-wave line sometimes used in a high-density antenna environment? [Johnson, '61]

30.39 It is stated that for a half-wave long transmission line, the magnitude of the voltage at the load is equal to the magnitude of the voltage at the source (voltage at the input of the line); that is, the load voltage (voltage at the output of the line) is independent of the load impedance! Determine when this is true. Why is a half-wave line sometimes used in a high-density antenna environment? If the voltage at the input has a nonzero resistance, is this statement valid? [Johnson, '61]

30.40 Why are transient tests more demanding on an antenna than sweep frequency tests?

30.41 The current at the input of a receiving loop antenna that is not electrically small is measured. This current is small. Does this imply that the current along the loop is negligible and hence the induced current is small? [Weeks, '64]

30.42 Starting from the basic definition

$$L_e = \frac{1}{I_o} \int_{\text{antenna}} I(z)\,dz$$

show that the effective length of a thin half-wave dipole with a sinusoidal current distribution is $\lambda/\pi$.

30.43E For an H-probe, determine whether placing a capacitor across the antenna leads will increase the operating frequency range of the probe.

30.44 A proposed RF current probe consists of an insulated wire in the shape of a squared-figure eight as shown in Figure 3. To measure the current, *I*, in a wire, the probe is placed near the wire as shown. It is stated that this probe is less likely to pick up distant stray magnetic fields than a simple circular-shaped probe. Determine whether there is any validity to this statement. Hint: carefully apply Lenz's law to each loop.

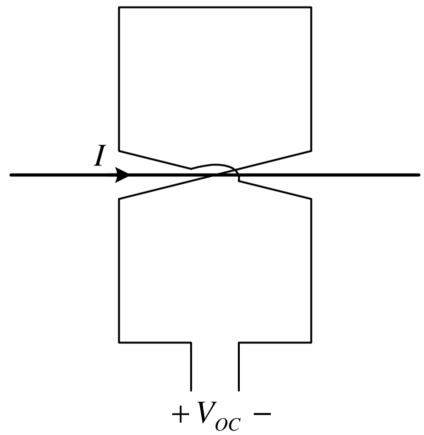


**Figure 3**

30.45C For small values of $l_{th}$, approximate the following equivalent length formula for sinusoidal current distribution along a thin dipole of total length $2l_{th}$:

$$L_e = \frac{\lambda}{\pi}\tan\left(\frac{\pi l_{th}}{\lambda}\right)$$

Plot the percent error associated with the approximation for $l_{th}/\lambda$ values ranging from 0.1 to 0.5.

30.46C For a sinusoidal current distribution along a thin dipole of total length less than or equal to $\lambda/2$, show that the dipole's effective length is

$$L_e = \frac{\lambda}{\pi}\left[1-\cos\left(\frac{2\pi l_{th}}{\lambda}\right)\right]$$

Then, plot the effective length versus $l_{th}$ using this equation and, on the same set of axes, plot the dipole's effective length that is based on a more sophisticated current distribution:

$$L_e = \frac{\lambda}{\pi}\tan\left(\frac{\pi l_{th}}{\lambda}\right)$$

Determine the maximum length $l_{th}$ so that the error between the results is less than 10%.

30.47 Determine the effective length of a small loop antenna with $N$ turns each of radius $a$.

30.48 Compare the open-circuit voltage across an E-field probe and an H-field probe if the length of the E-field probe is equal to the circumference of the loop of the H-field probe. Which voltage is larger and under what conditions? Assume that the probes are measuring the fields from a plane wave. What other practical factors should be considered when comparing the two probes?

30.49 Determine the relationship between the effective length and effective area for a receiver matched to its antenna.

30.50C Plot the inductance versus number of turns for a single-layer inductor constructed of 10 cm of #28 AWG around an air core. Determine where the maximum inductance occurs. Repeat for a 5 cm length of wire. Assume the turns of wire are tightly wound (but not touching).

30.51C Another equation used to estimate the inductance of a single-layer or multilayer air-core coil is

$$L = 0.885kDN^2 \times 10^{-6}$$

where $k$ is the inductance factor, $N$ is the number of turns, and $D$ is the average diameter of the coil. The inductance factor is a function of the ratio $D$ to the coil length, $l_{th}$. The approximate value for the inductance factor for several different ratios for single-layer and multilayer coils is given in Table 1. Compare this equation for the inductance of single and multilayer coils to that provided in this chapter. [Watt]

**Table 1**

| $D/l_{th}$ | $\approx k$ (single layer) | $\approx k$ (multilayer) |
|---|---|---|
| 1 | 0.74 | 0.4 |
| 2 | 1.2 | 0.7 |
| 3 | 1.4 | 0.95 |
| 5 | 1.8 | 1.3 |
| 7 | 2.1 | 1.5 |
| 10 | 2.3 | 1.8 |
| 20 | 2.6 | 2.3 |
| 30 | 3.1 | - |
| 50 | 3.2 | - |

30.52 It is stated that the maximum $Q$ for a single-layer air-core inductor occurs when the coil length is equal to the coil diameter (even though this may not always be practical). Determine the validity of this statement.

30.53 Sketch the magnetic flux density both inside and outside of a current-carrying air-core coil of length $l_{th}$, the same coil wrapped on a ferrite rod of length $l_{th}$, and the same coil wrapped on a ferrite rod of length $5l_{th}$. Then, sketch the flux density both inside and outside an open-circuited air-core coil, the same coil wrapped on a ferrite rod of the length $l_{th}$, and the same coil wrapped on a ferrite rod of length $5l_{th}$ when the coil is normally incident to a uniform magnetic field. Compare the field density inside the coil for all of these cases. Is the effective permeability equal to the apparent permeability?

30.54 Is the inductance of a one-loop inductor always less than a two-loop version (closely wound) constructed of the same wire length, gauge, and conductivity?

30.55E Show that if a loop is loaded with an impedance that is less than its self reactance, the loop's frequency dependence is reduced at a cost of reduced sensitivity. [Bronaugh]

30.56 For direction finding systems, would the loop or the rod antenna be more useful?

30.57 For short-range communications (but not direct line of sight), provide two disadvantages of a vertically oriented rod antenna.

30.58EC To "sniff-out" where RF radiation is leaking from a TV, a probe loop consisting of two three-inch diameter turns of insulated #20 wire is connected to the end of a piece of 50 Ω coaxial cable. The coax is then connected to the 50 Ω input of an oscilloscope. Discuss the effectiveness of this probe. What is the VSWR on the cable? [Hare]

30.59 Qualitatively explain how the voltage along a perfectly conducting antenna element can vary.

30.60 Derive the general Friis equation

$$\frac{P_{rec}}{P_{in}} = \left(\frac{\lambda_o}{4\pi r}\right)^2 G_{dt} G_{dr} \left(1 - |\rho_t|^2\right)\left(1 - |\rho_r|^2\right)$$

where $\rho_t$ is the reflection coefficient at the transmitting antenna and $\rho_r$ is the reflection coefficient at the receiving antenna. Show that the power reflection coefficients

$$|\rho_t|^2 = \rho_t \rho_t^* = \left(\frac{Z_{antt} - Z_{ot}}{Z_{antt} + Z_{ot}}\right)\left(\frac{Z_{antt}^* - Z_{ot}}{Z_{antt}^* + Z_{ot}}\right)$$

$$|\rho_r|^2 = \rho_r \rho_r^* = \left(\frac{Z_{lr} - Z_{antr}}{Z_{lr} + Z_{antr}}\right)\left(\frac{Z_{lr} - Z_{antr}^*}{Z_{lr} + Z_{antr}^*}\right)$$

are both one when the antennas are impedance matched.