Chapter 22: Electric Field Shielding

22.1 An object *A* in an electric field momentarily touches a grounded object. Qualitatively analyze the situation (from the standpoint of charge transfer) when the object *A* is a good conductor. Allow the contact time to vary from much less to much greater than the time constant associated with the charge transfer.

22.2 Repeat Problem 22.1 for an object *A* that is a good insulator.

22.3 Show that an electric field "ray" originating from a perfect conductor cannot terminate anywhere on this same conductor.

22.4 Show that in a system involving only two conductors (whether charged, grounded, or floating), electric field "rays" cannot both originate from and terminate on the same conductor.

22.5 Show by example that the surface charge along a charged perfect conductor can change sign.

22.6 Show by example that the surface charge along a charge-neutral perfect conductor can change sign.

22.7 Provide a situation where the potential of a single conducting body is zero yet the surface charge along it is not zero everywhere.

22.8 A positive volume charge is surrounded by a (charge-neutral) good-conducting spherical shell. Sketch the electric field distribution outside the shell, assuming the shell is isolated or far from other objects. The shell is then connected via a grounding strap to a very large ground plane. Again, sketch the electric field distribution outside the shell. The strap is then removed and the ground plane is moved far from the shell. Sketch the electric field distribution outside and near the shell.

22.9 A charge-neutral conductor is surrounded by (but insulated from) a spherical, floating, charge-neutral conducting shield with no openings. Outside this shield, a slowly varying voltage source is connected between another conductor and a large ground plane. As stated in this chapter, the mutual capacitance between this external conductor and the neutral conductor inside the shield is zero. What is the difference in potential between the shield and the conductor inside the shield? As the potential of the external conductor varies, does the potential of the conductor inside the shield vary? If so, why is the mutual capacitance between the conductors on both sides of the shield zero?

22.10 Referring to Figure 1, sketch the surface charge and electric field distributions if the total charge on conductor 2 is eight negative "units" (obtained from the ground plane). Conductors 1 and 3 are floating and charge neutral, and the ground plane is large. Assume there is some coupling between all of the conductors. Repeat the analysis if conductor 3 is connected to the ground plane (i.e., grounded).

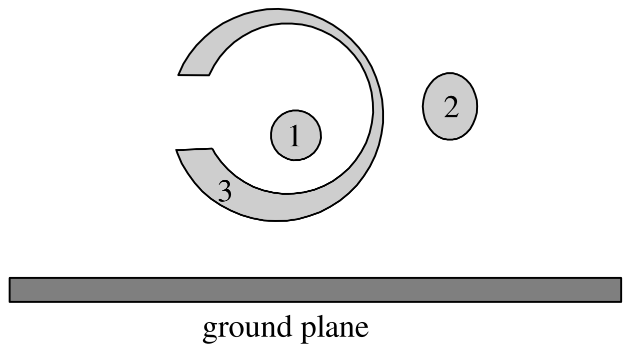

Figure 1

22.11 Referring to Figure 1, sketch the surface charge and electric field distributions if the total charge on conductor 3 (obtained from the ground plane) is eight negative “units.” Conductors 1 and 2 are floating and charge neutral, and the ground plane is large. Assume there is some coupling between all of the conductors. Repeat the analysis if conductor 1 is connected to the ground plane (i.e., grounded).

22.12 It is stated that the potential of a measurement device should be similar to the device being tested. Explain whether this statement is reasonable. If a trimmer capacitor is to be adjusted on a high-Q LC wave trap filter, what precautions should be taken?

22.13 For a spherical shell at high frequencies, can the thickness of the shell be much greater than the skin depth and the outer radius of the shell be electrically small?

22.14 The expression for the electric field at the center of the thin, spherical conducting shell of thickness Δ and conductivity σ was given in this chapter as

$$\vec{E}_{cnts} = 3E_e\left(\frac{j\omega\varepsilon_o}{\sigma}\right)\left(\frac{R}{2\Delta}\right)\hat{a}_z$$

Derive this expression by answering the following questions:

a) The sinusoidal steady-state potential outside a solid spherical object of conductivity σ, permittivity ε, and radius R when placed in a uniform electric field $\vec{E}(t) = E_e\cos(\omega t)\hat{a}_z$ is given by the expression

$$\Phi_o = -\mathrm{Re}\left[RE_e\cos\theta e^{j\omega t}\left\{\frac{r}{R} + \left[\frac{-\sigma + j\omega(\varepsilon_o - \varepsilon)}{\sigma + j\omega(2\varepsilon_o + \varepsilon)}\right]\left(\frac{R}{r}\right)^2\right\}\right]$$

First, from this expression, determine the electric field in the frequency domain outside of this solid spherical body. Second, by assuming the body is a very good conductor, approximate this electric field expression. Third, by using the boundary condition

$$\rho_S = D_{n2} - D_{n1} = \varepsilon_o E_{n2} - \varepsilon E_{n1}$$

where n subscript indicates the normal component, and assuming the electric field inside the body is small, show that the surface charge density, ρ_S, along the surface of the solid spherical body is given by

$$\rho_{Ss} = 3\varepsilon_o E_e \cos\theta e^{j\omega t}$$

b) Using the previous surface charge expression, determine the magnitude of the total charge induced on either hemisphere, or half, of the solid body. (The total charge on the entire spherical body should be zero since it is charge neutral.)
c) The applied field is varying; thus, the surface charge is also varying. Since $i = dq/dt$, in the frequency domain $I_s = j\omega Q_s$. Assume the charge distribution along the thin shell is the same as along the outer surface of the solid body. Determine the current passing in the shell of the spherical body in the frequency domain.
d) Near the equator of the spherical shell, assuming the shell's thickness $\Delta << R$, show that the dc or low-frequency (neglecting the skin effect) resistance of a hoop of the shell of thickness l_{th} is

$$R_{l_{th}} = \frac{l_{th}}{\sigma\Delta 2\pi R}$$

Clearly indicate the direction of the current through this segment. Then, using the previously derived current expression, determine the frequency-domain expression for the voltage drop across this hoop.
e) Because of symmetry, an equipotential contour must be along the equator of this sphere. Explain why this is true. Take the 0 V contour along this equator. Next, assume the electric field is approximately uniform near this equator (i.e., equipotentials near the equator are approximately parallel to the equator) and the field near the surface of the shell close to the equator is about equal to that at the center of the shell. Use the previously derived hoop voltage drop to obtain the desired expression for the electric field at the center of the thin conducting shell.

22.15 Estimate the strength of the electric field inside a typical human body from a cellular telephone. Assume an external, incident field strength of 10 V/m. State all assumptions.

22.16 The electric field inside the cavity of an insulating spherical shell of inner radius a, outer radius b, and relative permittivity ε_r when placed in a uniform electric field of strength E_o is

$$E_i = \frac{9\varepsilon_r}{9\varepsilon_r - 2(\varepsilon_r - 1)^2 \left(\frac{a^3}{b^3} - 1\right)} E_o$$

Determine an approximation for this equation when the thickness is small (but not zero) and the relative permittivity is not necessarily large.

22.17 The electric field inside the cavity of an insulating cylindrical shell of inner radius a, outer radius b, and relative permittivity ε_r when placed in a uniform electric field of strength E_o is

$$E_i = \frac{4\varepsilon_r}{4\varepsilon_r - (\varepsilon_r - 1)^2 \left(\frac{a^2}{b^2} - 1\right)} E_o$$

Determine an approximation for this equation when the thickness is small (but not zero) and the relative permittivity is not necessarily large.

22.18E Two closely space, perfectly conducting, circular plates or radius R, separated by a distance of d, are connected to a number of identical voltage sources, all equal to $v(t) = A\cos(\omega t)$. Free space exists between these two plates. These voltage sources, which are in parallel, are connected along the perimeter of the plates. As discussed in this chapter for two rectangular plates, neglecting edge effects, determine the expressions for the zero-order electric field between the plates, the surface charge density on both plates, and the surface current on either plate. From the zero-order electric field, determine the magnetic field between the plates. From this magnetic field, determine the corrected electric field between the plates that contains both the zero-order field and the first-order correction. As in this chapter, determine the ratio of the error electric field to the static, zero-order electric field. Under what conditions is the error field negligible?