Chapter 2: Decibel and Approximations

2.1 The signal level on a transmission line is 10 μW. Convert this absolute power level to dBm. Determine the voltage level in dBmV. State all assumptions.

2.2 For an AM signal to be "noise free," the signal-to-noise ratio must be 100. Convert this ratio to dB. State all assumptions.

2.3 A −7 dBm signal is at the input of two systems connected in cascade. The first system has a gain of 5 dB. The second system has a gain of 12 dB. Determine the absolute output power. State all assumptions.

2.4 Why and how did individuals use the logarithm to obtain the product of two numbers in the "old days" before the availability of inexpensive calculators?

2.5 If an amplifier's gain is given as 30 dB, is this a power or voltage gain?

2.6 Determine a simplified, approximate, and fractionless expression in dB for the following function:

$$H(\omega) = \frac{3\omega^3 + 4}{\omega + 2}$$

State all assumptions.

2.7 Determine a simplified, approximate, and fractionless expression in dB for the following function.

$$H(\omega) = \omega + \frac{4}{\omega^2 + 2}$$

State all assumptions.

2.8 Determine an approximate absolute value corresponding to 114 dB without using an electronic computing device (slider rulers, however, may be used).

2.9 Determine an approximate absolute value corresponding to 78 dB without using an electronic computing device.

2.10 Determine an approximate absolute value corresponding to 33 dB without using an electronic computing device.

2.11 A signal source designed for a 50 Ω load indicates a delivered power of 10 W. The actual load, however, is 100 Ω. What is the true absolute and dB equivalent power delivered to the load?

2.12C For each of the following functions, determine a "reasonable," nontrivial approximation for the function for small values of x of the form

$$a + bx + cx^2$$

(where a, b, and c are constants). Then, determine a similar approximation for large values of x (relative to 1), if possible, or a limiting value(s), if not possible. Plot the exact expression and each of the approximations on the same plot and plot the absolute percent difference between the exact expression and each of these approximations on another set of axes. Use a logarithmic scale for both axes for the percent difference plot. The variable x should range from x << 1 to x >> 1:

a) $7e^{2\sin(4x)}$
b) $2\cosh(3x)$
c) $\dfrac{x^3-2x^2}{\left(2x^2+1\right)^{\frac{3}{2}}}$
d) $\ln\left(3x+\sqrt{2x^2+3}\right)$

2.13C In reference to the approximation table provided in this chapter for the function # (provided by your instructor), plot the exact expression and each of the approximations on the same plot and plot the absolute percent difference between the exact expression and each of these approximations on another set of axes. Use a logarithmic scale for both axes for the percent difference plot. The range for the independent variable (e.g., x or n) should be such that all of the approximations can be checked.

2.14 In reference to the approximation table provided in this chapter, verify all of the Taylor series approximations for the function # (provided by your instructor) using the definition

$$f(x)=f(a)+(x-a)\frac{df(a)}{dx}+\frac{(x-a)^2}{2!}\frac{d^2f(a)}{dx^2}+\frac{(x-a)^3}{3!}\frac{d^3f(a)}{dx^3}+\dots$$

Finally, using some other nonseries definition for the function, verify all (if any) of the large value (or small value when appropriate) approximations provided for that function.

2.15C For each of the functions given in Table 1, plot the % error for both approximations on the same set of axes. Determine which of the approximations is "best." [Harris, '98]

Table 1

function	**approximation #1**	**approximation #2**
$\csc^{-1}(x)=\arcsin\left(\frac{1}{x}\right)$	$\frac{1}{x}+\frac{1}{6x^3}$ if $\lvert x\rvert >> 1$	$\frac{1}{x}\sqrt{\frac{3x^2}{3x^2-1}}$ if $\lvert x\rvert > 2$
$\tan^{-1}(x)=\arctan(x)$	$x-\frac{x^3}{3}$ if $\lvert x\rvert << 1$	$\frac{3x}{3+x^2}$ if $\lvert x\rvert \le 0.45$
$\sin^{-1}(x)=\arcsin(x)$	$x+\frac{x^3}{6}$ if $\lvert x\rvert << 1$	$x\sqrt{\frac{3}{3-x^2}}$ if $\lvert x\rvert \le 0.5$

$\sec(x)=\left[\cos(x)\right]^{-1}$	$1+\frac{x^2}{2}$ if $\lvert x\rvert<<1$	$\left(1-\frac{x^2}{3}\right)^{-\frac{3}{2}}$ if $\lvert x\rvert\le 0.9$
$\tan(x)$	$x+\frac{x^3}{3}$ if $\lvert x\rvert<<1$	$\frac{x}{1-\frac{x^2}{3}}$ if $\lvert x\rvert\le 0.6$
$\cot(x)=\left[\tan(x)\right]^{-1}$	$\frac{1}{x}-\frac{x}{3}$ if $\lvert x\rvert<<1$	$\frac{\left(1-\frac{x^2}{6}\right)^2}{x}$ if $\lvert x\rvert\le 0.5$