Chapter 11: Transient Behavior in the Frequency Domain

11.1C In the early part of this chapter, the magnitude of the Laplace transform at various frequencies was numerically studied for a triangle function. Repeat the entire numerical analysis and discussion for the following functions:

a) $\frac{1}{\sqrt{t+2}}u(t)$

b) $u(t-2)-u(t-4)$

The periods of the sinusoids used in the numerical analysis should have sufficient range to show accurately the amplitude spectrum of this time function. The parameter σ may be set to zero to illustrate more clearly the significance of the transform.

11.2C Repeat Problem 11.1 for all three of the following functions:

$$e^{-2t}u(t),\ e^{-20t}u(t),\ e^{-200t}u(t)$$

Then, compare the spectrums and discuss why the trend is reasonable.

11.3 Use the definition for the Laplace transform to determine the Laplace transform for the each of the following functions:

a) $(t-a)u(t)$

b) $tu(t-a)$

c) $e^{-at}u(t-b)$

d) $te^{-at}u(t)$

e) $\cosh(at)u(t)$

Verify that the expressions given in the Laplace transform table are correct. An integral table may be used.

11.4 Provide a partial check of the Laplace transform pair number # given in the full unabridged table in this book (# provided by the instructor) by using one or more of the transform properties and transforms provided in the abridged Table 1. The partial check may not consist of merely multiplying the function or transform by a constant, setting a variable to zero, to infinity, or to another constant, or splitting or combining the function or transform (e.g., linearity). Do not use the definition for the Laplace transform or the inverse transform as the check.

Table 1

Time Function	Laplace Transform	Time Function	Laplace Transform
$f(t-a)u(t-a)$	$e^{-as}F(s)$	$\delta(t)$	1
$e^{-at}f(t)$	$F(s+a)$	$u(t)$	$\frac{1}{s}$
$tf(t)$	$-\frac{dF(s)}{ds}$	$\sqrt{t}u(t)$	$\frac{1}{2s}\sqrt{\frac{\pi}{s}}$
$\frac{f(t)}{t}$	$\int_s^\infty F(s)ds$	$\frac{1}{\sqrt{t}}u(t)$	$\sqrt{\frac{\pi}{s}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0^-)$	$e^{-at}u(t)$	$\frac{1}{s+a}$
$\int_{0^-}^t f(t)dt$	$\frac{F(s)}{s}+\frac{\int_{0^-}^{0^+} f(t)dt}{s}$	$\sin(at)u(t)$	$\frac{a}{s^2+a^2}$
$\int_0^t \frac{f(t)}{t}dt$	$\frac{1}{s}\left[\int_s^\infty F(s)ds\right]$	$\cos(at)u(t)$	$\frac{s}{s^2+a^2}$
$f(t)\sin(at)$	$\frac{1}{2j}\left[F(s-ja)-F(s+ja)\right]$	$\sinh(at)u(t)$	$\frac{a}{s^2-a^2}$
$f(t)\cos(at)$	$\frac{1}{2}\left[F(s-ja)+F(s+ja)\right]$	$\cosh(at)u(t)$	$\frac{s}{s^2-a^2}$
$f(t)\sinh(at)$	$\frac{1}{2}\left[F(s-a)-F(s+a)\right]$		
$f(t)\cosh(at)$	$\frac{1}{2}\left[F(s-a)+F(s+a)\right]$		

11.5 By using any of the transform properties given in Table 2, verify the Laplace transform of the periodic or other function # given in the unabridged table in the book (# provided by your wise instructor). For the nonperiodic functions it may be necessary to sum several or many periodic functions or use various infinite series.

Table 2

Time Function	Laplace Transform	Comments
$f(t)=f(t+T)$	$\frac{1}{1-e^{-Ts}}\int_0^T f(t)e^{-st}dt$	$f(t)$ is a periodic function with a period of T

$f(t) = -f\left(t + \frac{T}{2}\right)$	$\frac{1}{1+e^{-\frac{Ts}{2}}} \int_0^{T/2} f(t)e^{-st}dt$	$\lvert f(t) \rvert$ is a periodic function with a period of $T/2$ (half-wave symmetry)
$f(t)\sum_{n=0}^{\infty}(-1)^n u\left(t - \frac{nT}{2}\right)$ where $f(t) = -f\left(t + \frac{T}{2}\right)$	$\frac{F(s)}{1-e^{-\frac{Ts}{2}}}$	half-wave rectification of $f(t)$ (with half-wave symmetry) where $F(s)$ is the transform of the entire periodic waveform (not just one period)
$\lvert f(t) \rvert$ where $f(t) = -f\left(t + \frac{T}{2}\right)$	$F(s)\left(\frac{1+e^{-\frac{Ts}{2}}}{1-e^{-\frac{T}{2}s}}\right) = F(s)\coth\left(\frac{Ts}{4}\right)$	full-wave rectification of $f(t)$ (with half-wave symmetry) where $F(s)$ is the transform of the entire periodic waveform (not just one period)

11.6 Verify the transform pair given for the second derivative of the function

$$y(t) = 3te^{-4t}\sin(20t)u(t)$$

$$\frac{d^2y(t)}{dt^2} \Leftrightarrow s^2\left[\frac{2(20)(s+4)}{(s+4)^2+(20)^2}\right]$$

by actually taking the second derivative of $y(t)$ and then taking its transform.

11.7 For the Laplace transform pair number # given in the table (# provided by the instructor), use the initial-value theorem and the final-value theorem (if it satisfies the negative, nonzero pole requirement) to determine the initial and final values of the time function from its transform. Then, check these results by actually determining the initial and final values from the time function.

11.8 Sketch and label each of the following functions:

a) $g(x) = (2x-1)u(x) + u(x+3) - 4u(6-3x)$

b) $h(t) = \sin(2t)u(t-3) - \sin(2t)u(t-6) - 3u(2t+1) + e^{-0.1t}u(6t-9)$

c) $s(t) = u(t+2) - u(t-1) + 3t(3t-1) - (-5t+6)\left[u(t-4) - u(t-7)\right]$

d) $m(t) = t^2u(3-2t) - u(2t-1) + \cos(2\pi t)u(t)$

e) $q(y) = yu\left(e^y - 1\right)$

f) $x(t) = t^2u\left(t^2 - 1\right)$

When determining the total output signal by summing the individual functions, the value of each individual function should be examined immediately before and after any discontinuity. When summing functions that are discontinuous, the output *can* be discontinuous.

11.9 Using step functions, write a simple expression for the waveforms shown in Figure 1, Figure 2, Figure 3, and Figure 4. Do not simplify the expression.

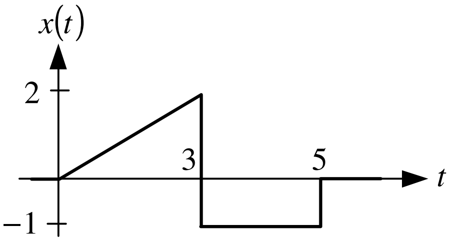

Figure 1

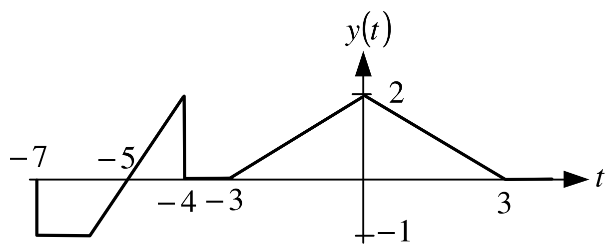

Figure 2

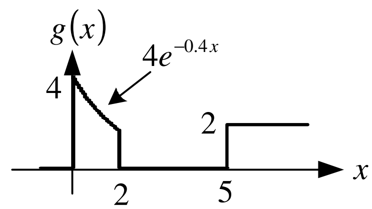

Figure 3

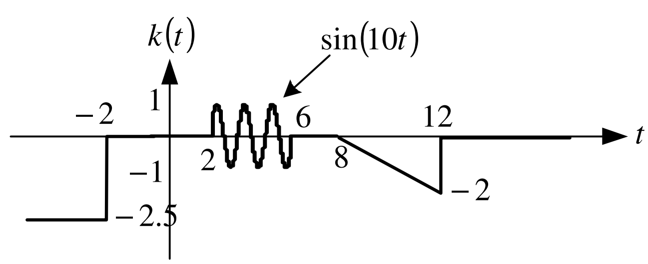

Figure 4

11.10 For the functions shown in Figure 1, Figure 2, Figure 3, and Figure 4, sketch their derivatives with respect to time without using any unit steps and without writing out an expression for the function. Then, write a mathematical expression for the functions using unit steps and then analytically determine its derivative from this expression. Show that the graphical and analytical results are equal.

11.11 Evaluate each of the following integrals involving the step function:

a) $x(t)=\int_{-\infty}^{t} e^{-2t}u(t-1)u(3-t)\,dt$

b) $y(t)=\int_{-\infty}^{\infty} \cos(x)u(x-1)u(t+x+2)\,dx$

c) $z(t)=\int_{-20}^{t} (3\lambda+1)u(\lambda+1)u(\lambda+2-t)\,d\lambda$

d) $g(t)=\int_{2}^{-t} 3yu(2y+1)u(t+y-2)\,dy$

e) $h(t)=\int_{-2}^{10} 3u(2x+1)u(t-x+5)\,dx$

11.12 For the periodic or periodic-like function # provided in the Laplace transform table (# provided by your competent instructor), describe the function as a function of an infinite number of step functions in a summation form.

11.13 Use the Laplace transform to verify the expressions given in this chapter for the impulse and step response for the voltage across the inductor for the series *RL* circuit.

11.14 Use the Laplace transform to verify the expressions given in this chapter for the impulse and step response for the voltage across the resistor for the series *RC* circuit.

11.15 Use the Laplace transform to verify the expressions given in this chapter for the impulse and step response for the voltage across the capacitor for the series *RC* circuit.

11.16C Referring to Figure 5 where $v_s(t)=A\cos(\omega t)$, using Laplace transforms determine the voltage across the inductor in the time domain. Then, check the result by allowing the source frequency to be equal to zero. Then, check the result by allowing the time to become sufficiently large so the transient part of the solution is negligible.

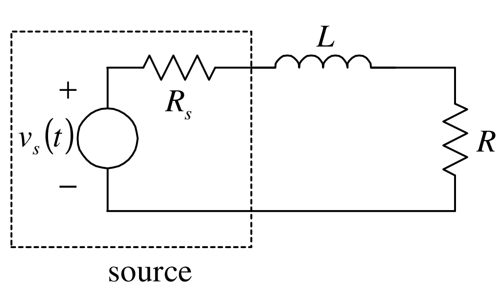

Figure 5

11.17C Repeat Problem 11.16 for the voltage across *R* if the inductor is replaced with a capacitor, *C*.

11.18C Repeat Problem 11.16 for the voltage across *C* if the inductor is replaced with a capacitor, *C*.

11.19 For the circuit given in Figure 6, determine and then sketch the voltage across the capacitor for each of the following input signals. Assume the initial voltage across the capacitor is zero volts. Is the derivative of the step response the impulse response?

$(a)\ v_s(t) = \delta(t)$

$(b)\ v_s(t) = u(t)$

$(c)\ v_s(t) = tu(t)$

$(d)\ v_s(t) = t^2u(t)$

$(e)\ v_s(t) = e^{-t}u(t)$

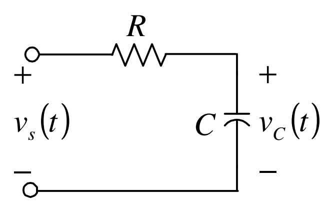

Figure 6

11.20E Using the Laplace transform, determine the time-domain voltage for the first three periods across the inductor in a simple series *RL* circuit. The source voltage, which is in series with the *R* and the *L*, is a square wave with a period of *T*, dc offset of zero, and peak-to-peak amplitude of two. If *R* >> *L*, sketch this voltage waveform for several periods and explain why this circuit is referred to as a differentiating circuit. Compare these results with those provided in the *Transient Behavior in the Time Domain* chapter.

11.21C Repeat the analysis given in the voltage-zapper discussion in this chapter for the high-voltage impulse generator shown in Figure 7. For the numerical portion of the analysis, let R_1 = 100 Ω, R_2 = 1 kΩ, C_1 = 10 µF, C_2 = 1 µF, and $v_{C1}(0)$ = 30 kV.

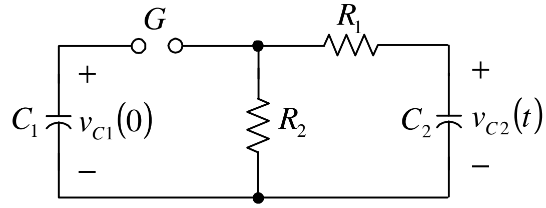

Figure 7

11.22C Repeat the analysis given in the voltage-zapper discussion in this chapter for the high-voltage impulse generator shown in Figure 8. For the numerical portion of the analysis, let R_1 = 100 Ω, R_2 = 1 kΩ, R_3 = 1 kΩ, C_1 = 10 μF, C_2 = 1 μF, and $v_{C1}(0)$ = 30 kV.

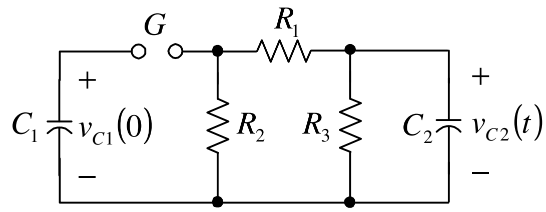

Figure 8

11.23C Repeat the analysis given in the voltage-zapper discussion in this chapter for the high-voltage impulse generator shown in Figure 9. For the numerical portion of the analysis, let R_1 = 100 Ω, R_2 = 1 kΩ, L = 1 μH, C_1 = 10 μF, C_2 = 1 μF, and $v_{C1}(0)$ = 30 kV.

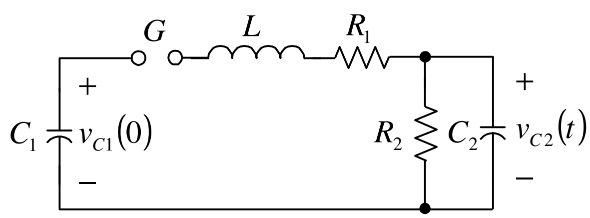

Figure 9

11.24C Repeat the entire "blimp amplitude" analysis given in this chapter (including the approximations and numerical analysis) for the given transfer function corresponding to the transmission into a shunt capacitive load:

$$H(s) = \frac{1}{1 + \frac{sCZ_o}{2}}$$

11.25C Repeat the entire "blimp amplitude" analysis given in this chapter (including the approximations and numerical analysis) for the given transfer function corresponding to the reflection from series inductive loading (i.e., an inductor connected in series between two transmission lines):

$$H(s) = \frac{-1}{1 + \frac{2sL}{Z_o}}$$

Select an appropriate value for L for the approximation to satisfy any conditions required in the approximation.

11.26EC If $R = 1$ kΩ and $C = 0.01$ μF, determine and plot (on the same set of axes) the step response for each of the RC circuits given in Figure 10 using Laplace transforms. Explain how these circuits shift the phase of the input signal. Which circuit has the greatest phase shift?

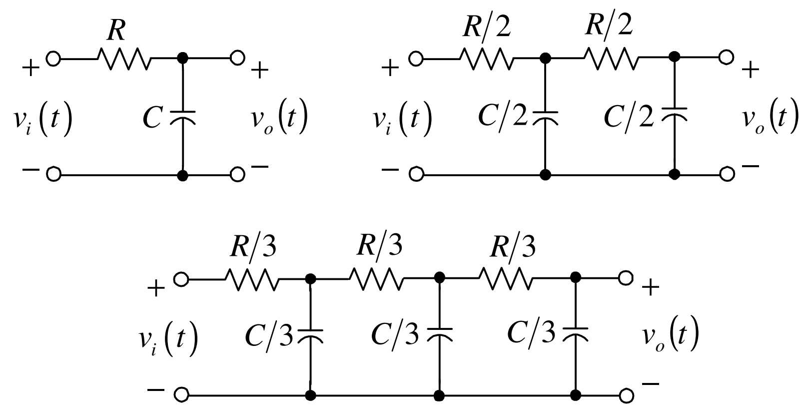

Figure 10

11.27EC If $R = 1$ kΩ and $C = 0.01$ μF, using Laplace transforms, determine and plot (on the same set of axes) the impulse response for each of the circuits given in Figure 10.

11.28EC Repeat Problem 11.26 but interchange the positions of the resistor and the capacitors.

11.29EC Repeat Problem 11.26 but replace all of the capacitors with inductors. Let L = 10 mH.

11.30EC Repeat Problem 11.26 but interchange the positions of the resistors and the capacitors and then replace all capacitors with inductors. Let L = 10 mH.

11.31EC If $R = 1$ kΩ, $C = 0.01$ μF, and $L = 10$ mH, determine and plot both the impulse and step responses for the circuit shown in Figure 11 using Laplace transforms. Sketch the Bode magnitude plot of the voltage gain as a function of the variables R, L, and C for this circuit. Why is this circuit sometimes referred to as a high-pass filter?

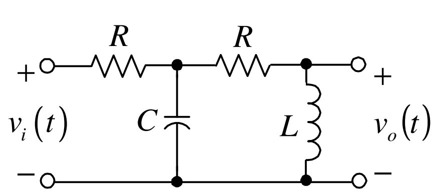

Figure 11

11.32 For the circuit given in Figure 11, determine the transfer function corresponding to the ratio of the output voltage to the input voltage. If the initial current through the inductor is $i(0)$ (directed upward), the initial voltage across the capacitor is $v(0)$ (positive polarity at the resistor node), and the transform of the input voltage is $V_i(s)$, determine the frequency-domain expression for the voltage across the capacitor and the current through the inductor. Do not substitute any values for R, L or C. Use the initial-value theorem to check these initial values assuming that the input signal is not impulsive. Is the transfer function any value in determining the voltage across the inductor?

11.33 Referring to Figure 12, if the switch closes at $t = 0$ (with zero initial current through the inductor) and reopens at $t = t_x$, determine the voltage $v_x(t)$ for $t > 0$ using the Laplace transform technique. If L and V_s are fixed in value, should the resistances be small or large to decrease the amplitude of any impulses or pseudo impulses in the voltage?

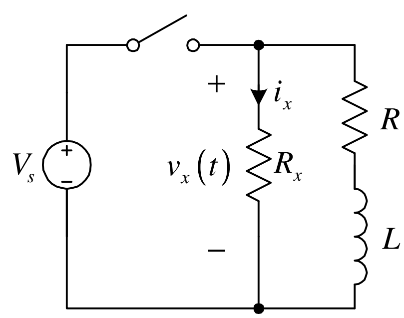

Figure 12

11.34C Continue the half-wave rectifier analytical and numerical analysis given in the later part of this chapter for the time interval $T \le t \le 2T$.

11.35 Working in the frequency domain, determine $I(s)$ and $i(t)$ for the circuit given in Figure 13 assuming that the initial current through the inductor is $i(0)$.

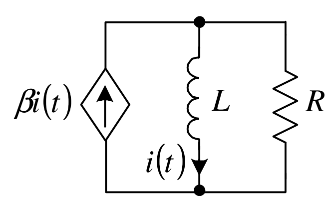

Figure 13

11.36 Working in the frequency domain, determine $I(s)$ and $i(t)$ for the circuit given in Figure 14 assuming that the initial voltage across the capacitor is $v(0)$.

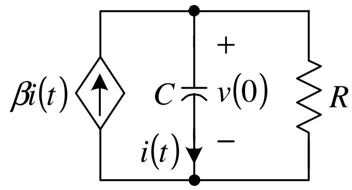

Figure 14

11.37C Determine the expression for the current $i(t)$ for the circuit given in Figure 15 if the supply voltage is

$$v_s(t) = mt\left[u(t) - u(t-a)\right] \quad a > 0$$

Assume the initial current through the inductor is zero. Then, plot this current if R_1 = 10 Ω, R_2 = 1 kΩ, R_3 = 1 Ω, L = 1 μH, and $m = a = \tau/10$, τ, and 10τ where τ is the time constant for this circuit.

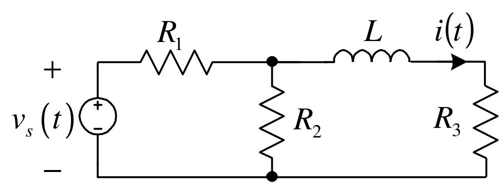

Figure 15

11.38 Determine the expression for the output voltage in the time domain for the circuit given in Figure 16, which is a model for a microphone. Assume that the input voltage is

$$v_i(t) = A\cos(\beta t)u(t)$$

and the initial voltage across each capacitor is zero. Then, determine the steady-state version of this expression by allowing the transients to die off.

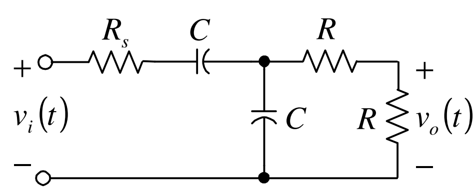

Figure 16