CHME420
Spring 2015
Exam 1
04/30/15 1:20 PM
Time Limit: 125 Minutes

Name (Print): ____________________

This exam contains 11 pages and 5 problems. Check for missing pages. Put your initials on the top of every page in case they become separated.

The exam is open book and notes. No cell phone use allowed during the exam.

You are required to show your work on each problem. The following rules apply:

- **Read the problem statements carefully**. Do what is requested in each problem. Show your work at every step to communicate your knowledge and thinking process.
- **Organize your work** neatly and coherently in the space provided. Scattered work without a clear order and difficult to understand will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive substantial partial credit. A correct answer accompanied randomly by an incorrect one will undergo a deduction.
- If you need more space, request extra blank pages from the instructor. Label each additional blank page with your name, the problem number, and the page number for that particular problem. Place your additional pages in the right order and staple them together.

Do not write in this table:

Problem	Points	Score
1	20	
2	25	
3	25	
4	15	
5	15	
Total:	100	

1. (20 points) For fully developed laminar flow in a thin slit with constant heat flux (q_0) at the walls, the thermally fully developed temperature profile is given by:

$$\Theta(\breve{x}, \breve{z}) = \frac{3}{2}\breve{z} + \frac{3}{4}\breve{x}^2 - \frac{1}{8}\breve{x}^4 - \frac{39}{280}$$

$$\Theta = \frac{T - T_1}{q_0 B/k}, \quad \breve{x} = \frac{x}{B}, \quad \breve{z} = \frac{kz}{\rho \hat{C}_p v_{z,max} B^2}$$

See problem 10B.7 in your textbook for the definition of all symbols and for a schematic of the problem. The velocity profile is given by:

$$v = v_{z,max}\left[1 - \left(\frac{x}{B}\right)^2\right]$$

Using the frame of reference and the schematic of Problem 10B.7 do the following:

(a) (4 points) Calculate the arithmetic mean temperature ($< T >$) over a cross-section as a function of $\breve{z}$.

(b) (4 points) Write the integral expression for the bulk or "cup-mixing" temperature (T_b) as a function of $\breve{z}$. Plug in the expressions for the velocity and the temperature profiles. Do not solve the integral (straightforward but time consuming). The solution of the integral is given by:

$$T_b = T_1 + \frac{3}{2}\breve{z}\frac{q_0 B}{k}$$

(c) (4 points) Calculate the temperature at the wall of the slit.

(d) (4 points) Calculate the Nusselt number.

(e) (4 points) An experiment uses a thermometer to measure the temperature at the center of the slit. Using dimensionless temperature variables (Θ), calculate the percent error if we use this centerline temperature as an estimate for $\Theta_b = (T_b - T_1)/(q_0 B/k)$ at $\breve{z} = 1$. Would we be under- or over- estimating the bulk temperature? What happens to the error as we move farther down the pipe?

(f) (4 points (bonus)) Why is your answer in Part d different than that in entry (L) of Table 14.2-2 of BSL? Hint: Pay attention to the title and notes in the table.

2. (25 points) In class we developed an equation for the temperature profile in slit flow accounting for viscous heating (see §10.4 for details and schematic). The top plate moves at 10 m/s, the gap between the plates is 0.10 m, the temperature of the bottom plate is 295 K, and the temperature of the top plate is 305 K. Do the following:
   (a) (10 points) Derive an expression for the location of the maximum temperature in the liquid.
   (b) (10 points) Calculate the location of the maximum temperature of the liquid (if it exists) and the value of the maximum temperature for water and glycerol. You may assume that the properties of the fluids are the same as those at the average temperature of the plates. In which case is viscous heating more significant? Why?
   (c) (5 points) What is the maximum value of the Brinkman number at which no maximum in the temperature is observed? In other words, what is the maximum value of the Brinkman number for which the maximum temperature is 305 K?

3. (25 points) Use an energy shell balance to find the temperature profile in a cylindrical fin of radius R and length L and for which it can be assumed that h (the heat transfer coefficient) is small and constant and that $L >> R$ (both of these assumptions together allow you to use the "fin approximation"). The temperature of the wall of the finned surface is T_w and the ambient temperature is T_a. Assume that the heat losses from the end of the fin can be neglected.

   (a) (4 points) Draw a schematic of the problem which includes your frame of reference and the various parameters in the problem.

   (b) (10 points) Perform the energy shell balance to obtain a differential equation relating the temperature to the axial coordinate of the fin. State the appropriate boundary conditions.

   (c) (4 points) Non-dimensionalize the differential equation and the boundary conditions using L as the characteristic length and the following definition for the dimensionless temperature:

$$\Theta = \frac{T - T_a}{T_w - T_a}$$

   (d) (2 points) What is the solution to the non-dimensional boundary value problem? You should not need to solve for it in this part. If you think you have to §C.1 in the textbook can be useful.

   (e) (5 points) The "fin approximation" allowed the reduction of the dimensionality of the problem from two dimensions to one. What would be the problem with the approach if h is large relative to k?

4. (15 points) In class we developed the flux and temperature profiles for a cladded nuclear fuel sphere as shown in Figure 10.3-1 in the textbook. At a particular power plant the fuel pellets have a radius of 0.20 m for the cladding and 0.10 m for the fuel. The heat generation at the center of the fuel rod is 1000.0 kW/m^3 and the dimensionless constant b is 1.5. The temperature at the surface of the cladding is 373 K and the thermal conductivities of fuel and cladding are 200 and 400 W/(m · K). Calculate the rate of heat release from a single pellet in kW.

5. (15 points) Water at a bulk temperature of T_i flows through a copper (thermal conductivity k_c) pipe of length L and with R_i and R_p inner and outer radii, respectively. It is desired to insulate (insulation radius R_o) the pipe with a material that has an electrical conductivity of k_i. The heat transfer coefficient inside and outside the pipe (or insulation) are h_i and h_o. The temperature outside the pipe is T_o.

   (a) (10 points) Write an expression (do not plug in any numbers) for the heat loss across the pipe, including the insulating layer.

   (b) (5 points) As insulation material is added, two competing processes occur. First, the surface area for heat transfer increases, potentially increasing the heat loss. Second, the insulating properties of the outer layer improve relative to the copper pipe, potentially decreasing the heat loss. This two processes give rise to a critical radius of insulation at which heat losses are a maximum. Suggest situations in which you would want to operate below and above the critical radius. Mention at least one for each and don't restrict yourself to situations in which the material to be insulated is a pipe.

   (c) (5 points (bonus)) Using the expression from Part (a) derive the critical radius of insulating material at which heat loss in the system is a maximum.