CHME420
Spring 2015
Exam 1
04/30/15 1:20 PM
Time Limit: 125 Minutes

Name (Print): ____________________

This exam contains 11 pages and 5 problems. Check for missing pages. Put your initials on the top of every page in case they become separated.

The exam is open book and notes. No cell phone use allowed during the exam.

You are required to show your work on each problem. The following rules apply:

- **Read the problem statements carefully**. Do what is requested in each problem. Show your work at every step to communicate your knowledge and thinking process.
- **Organize your work** neatly and coherently in the space provided. Scattered work without a clear order and difficult to understand will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive substantial partial credit. A correct answer accompanied randomly by an incorrect one will undergo a deduction.
- If you need more space, request extra blank pages from the instructor. Label each additional blank page with your name, the problem number, and the page number for that particular problem. Place your additional pages in the right order and staple them together.

Do not write in this table:

Problem	Points	Score
1	15	
2	35	
3	25	
4	15	
5	10	
Total:	100	

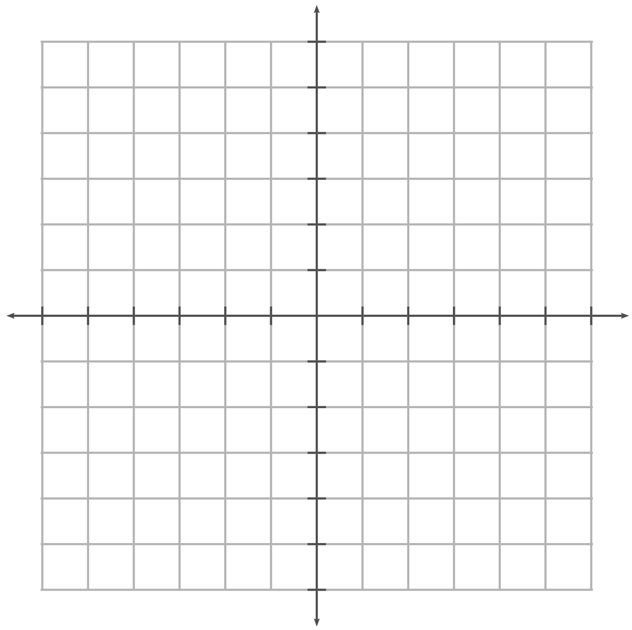

Figure 1: Cartesian axes for Question 1.

1. (15 points) Sketch the following flow field in the figure provided assuming that the flow is incompressible (make sure to label your axes):

$$v_x = -\frac{1}{2}x$$
$$v_y = 0$$
$$v_z = \frac{1}{2}z$$

At the points (x, z): (2,0), (-2,0), (0,2), (0,-2), (4,0), (-4,0), (2,2), (2,-2), (-2,2), and (-2,-2). Given this flow field, obtain the total momentum flux tensor $\underline{\underline{\phi}}$. You may omit the pressure terms. Show your work and state which components of the tensor are zero.

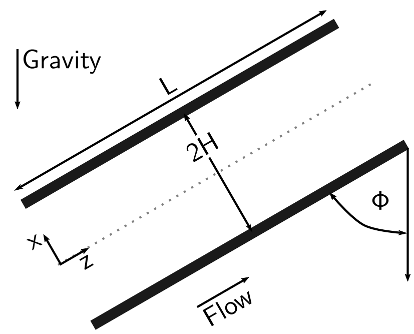

Figure 2: Inclined slit with pressure-driven flow.

2. (35 points) Figure 2 shows a pressure-driven flow of an incompressible, Newtonian liquid between two parallel plates (width B) separated by a gap $2H$. The slit is inclined from the vertical by an angle ϕ. The pressure at $z = 0$ is p_0 and the pressure at point $z = L$ is p_L. Assume the flow is a steady state to do the following:
   (a) (10 points) Suggest postulates and boundary conditions for the flow.
   (b) (15 points) Use a shell balance to obtain a differential equation that could be solved to obtain the velocity profile of the liquid.
   (c) (5 points) Solve for the velocity profile.
   (d) (5 points) Write expressions for the volumetric flow rate and the force on the upper wall.

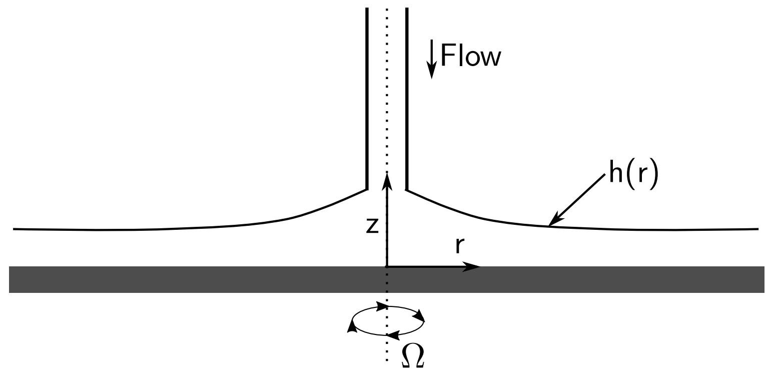

Figure 3: A lubricating liquid is being delivered to a rotating disk.

3. (25 points) A spinning disk is coated with a lubricant as shown in Figure 3. As a first approximation, the flow obeys the following differential equation:

$$-\rho r\Omega^2 = \mu\frac{\partial^2 v_r}{\partial z^2} \tag{1}$$

And boundary conditions:

$$\frac{\partial v_r}{\partial z} = 0 \text{ @ } z = h(r)$$
$$v_r = 0 \text{ @ } z = 0$$

Do the following:

(a) (4 points) State which variables and parameters does the velocity in the r direction depends on. Your answer should be in the form $v_r = v_r(...)$.

(b) (5 points) Non-dimensionalize Equation 1 and the boundary conditions by using the height $h(r)$ as the characteristic length. Show every step of your work. For this part you may treat $h(r)$ and r as constants. Your characteristic velocity should appear naturally to yield the non-dimensional equation:

$$-1 = \frac{\partial^2 \breve{v}_r}{\partial \breve{z}^2} \tag{2}$$

Write the expression for the non-dimensional velocity, $\breve{v}_r$

(c) (4 points) State which variables and parameters does $\breve{v}_r$ depend on. Your answer should be in the form $\breve{v}_r = \breve{v}_r(...)$.

(d) (5 points) Solve the non-dimensional equation subject to the non-dimensional boundary conditions to obtain the profile $\breve{v}_r(\breve{z})$.

(e) (3 points) Convert your non-dimensional velocity profile into a dimensional equation relating v_r, z, and any other parameters/variables of the model.

(f) (4 points) Calculate the volumetric flow rate of the fluid using the dimensional velocity profile.

4. (15 points) A flat plate is suddenly placed in motion with $v_x = 10\,\text{m/s}$ at $t = 0$. Glycerol at room temperature ($\rho = 1261\,\text{kg/m}^3$ and $\mu = 1.412\,\text{Pa}\cdot\text{s}$) is located on top of the plate. How long does it take for the momentum transport to penetrate $1\,\text{m}$ into the fluid. Perform the same calculation for room temperature water. How does the difference in viscosity and density affect the motion of the fluids?

5. (10 points) In class we saw how to calculate the friction factor for laminar flow in the pipe. We want to repeat this exercise for a duct with a cross-section shaped as an equilateral triangle. The length of the side of a triangle is a and the length of the duct is L. The friction factor is defined as:

$$F_k = A_w(KE)f$$

where A_w is the wetted area of the pipe (note this is not the cross-sectional area). F_k is the force due to the fluid motion and a force balance on the conduit yields:

$$F_k = A_c(\mathscr{P}_0 - \mathscr{P}_L)$$

where A_c is the cross sectional area of the pipe. The area of an equilateral triangle is $A_c = \sqrt{3}a^2/4$. It has been found that the average velocity inside the duct is given by:

$$< v_z >= \frac{a^2(\mathscr{P}_0 - \mathscr{P}_L)}{80\mu L}$$

Find a correlation between the friction factor and the Reynolds number for this system. See Chapter 6 for more details if needed.