Solutions to Even numbered problems in Chapters 5-7

CHAPTER 5 (Sections 5.1-5.8)

5-2. 1) The parameter of interest is the difference in breaking strengths, $\mu_1 - \mu_2$ and $\Delta_0 = 10$

2) $H_0: \mu_1 - \mu_2 = 10$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 > 10$ or $\mu_1 > \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 > z_\alpha = 1.645$

7) $\bar{x}_1 = 162.5$ $\bar{x}_2 = 155.0$ $\delta = 10$

$\sigma_1 = 1.0$ $\sigma_2 = 1.0$

$n_1 = 10$ $n_2 = 12$

$$z_0 = \frac{(162.5 - 155.0) - 10}{\sqrt{\frac{(1.0)^2}{10} + \frac{(1.0)^2}{12}}} = -5.84$$

8) Since -5.84 < 1.645 do not reject the null hypothesis and conclude there is insufficient evidence to support the use of plastic 1 at $\alpha = 0.05$.

5-4. $\bar{x}_1 = 30.87$ $\bar{x}_2 = 30.68$

$\sigma_1 = 0.10$ $\sigma_2 = 0.15$

$n_1 = 12$ $n_2 = 10$

a) 90% two-sided confidence interval:

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(30.87 - 30.68) - 1.645\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}} \le \mu_1 - \mu_2 \le (30.87 - 30.68) + 1.645\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}$$

$$0.0987 \le \mu_1 - \mu_2 \le 0.2813$$

We are 90% confident that the mean fill volume for machine 1 exceeds that of machine 2 by between 0.0987 and 0.2813 fl. oz.

b) 95% two-sided confidence interval:

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(30.87 - 30.68) - 1.96\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}} \le \mu_1 - \mu_2 \le (30.87 - 30.68) + 1.96\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}$$

$$0.081 \le \mu_1 - \mu_2 \le 0.299$$

We are 95% confident that the mean fill volume for machine 1 exceeds that of machine 2 by between 0.081 and 0.299 fl. oz.

Comparison of parts a and b:

As the level of confidence increases, the interval width also increases (with all other values held constant).

c) 95% upper-sided confidence interval:

$$\mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_\alpha \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\mu_1 - \mu_2 \le (30.87 - 30.68) + 1.645\sqrt{\frac{(0.10)^2}{12} + \frac{(0.15)^2}{10}}$$

$$\mu_1 - \mu_2 \le 0.2813$$

With 95% confidence, we bellieve the fill volumne for machine 1 exceeds the fill volume of machine 2 by no more than 0.2813 fl. oz.

5-6. $\bar{x}_1 = 89.6 \quad \bar{x}_2 = 92.5$

$\sigma_1^2 = 1.5 \quad \sigma_2^2 = 1.2$

$n_1 = 15 \quad n_2 = 20$

a) 95% confidence interval:

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(89.6 - 92.5) - 1.96\sqrt{\frac{1.5}{15} + \frac{1.2}{20}} \le \mu_1 - \mu_2 \le (89.6 - 92.5) + 1.96\sqrt{\frac{1.5}{15} + \frac{1.2}{20}}$$

$$-3.684 \le \mu_1 - \mu_2 \le -2.116$$

With 95% confidence, we believe the mean road octane number for formulation 2 exceeds that of formulation 1 by between 2.116 and 3.684.

b) 1) The parameter of interest is the difference in mean road octane number, $\mu_1 - \mu_2$ and $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_\alpha = -1.645$

7) $\bar{x}_1 = 89.6 \quad \bar{x}_2 = 92.5$

$\sigma_1^2 = 1.5 \quad \sigma_2^2 = 1.2$

$n_1 = 15 \quad n_2 = 20$

$$z_0 = \frac{(89.6 - 92.5) - 0}{\sqrt{\frac{(1.5)^2}{15} + \frac{(1.2)^2}{20}}} = -7.254$$

8) Since $-7.25 < -1.645$ reject the null hypothesis and conclude the mean road octane number for formulation 2 exceeds that of formulation 1 using $\alpha = 0.05$.

c) P-value = $P(z \le -7.25) = 1 - P(z \le 7.25) = 1 - 1 = 0$

5-8. 95% level of confidence, E = 1, and $z_{0.025} = 1.96$

$$n \cong \left(\frac{z_{0.025}}{E}\right)^2\left(\sigma_1^2+\sigma_2^2\right)=\left(\frac{1.96}{1}\right)^2(1.5+1.2) = 10.37,\ n = 11,\text{ use } n_1 = n_2 = 11$$

5-10.

Catalyst 1	Catalyst 2
$\bar{x}_1 = 65.22$	$\bar{x}_2 = 68.42$
$\sigma_1 = 3$	$\sigma_2 = 3$
$n_1 = 10$	$n_2 = 10$

a) 95% confidence interval on $\mu_1 - \mu_2$, the difference in mean active concentration

$$\left(\bar{x}_1-\bar{x}_2\right)-z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \le \mu_1-\mu_2 \le \left(\bar{x}_1-\bar{x}_2\right)+z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

$$(65.22-68.42)-1.96\sqrt{\frac{(3)^2}{10}+\frac{(3)^2}{10}} \le \mu_1-\mu_2 \le \left(65.22-68.42\right)+1.96\sqrt{\frac{(3)^2}{10}+\frac{(3)^2}{10}}$$

$$-5.83 \le \mu_1-\mu_2 \le -0.57$$

We are 95% confident that the mean active concentration of catalyst 2 exceeds that of catalyst 1 by between 0.57 and 5.83 g/l.

b) Yes, since the 95% confidence interval did not contain the value 0, we would conclude the mean active concentration depends on the choice of catalyst.

5-12. 1) The parameter of interest is the difference in mean active concentration, $\mu_1 - \mu_2$

2) $H_0: \mu_1-\mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1-\mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1-\bar{x}_2)-\Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$

7) $\bar{x}_1 = 750.2$ $\bar{x}_2 = 756.88$ $\delta = 0$

$\sigma_1 = 20$ $\sigma_2 = 20$

$n_1 = 15$ $n_2 = 8$

$$z_0 = \frac{(750.2-756.88)-0}{\sqrt{\frac{(20)^2}{15}+\frac{(20)^2}{8}}} = -2.385$$

8) Since $-2.385 < -1.96$ reject the null hypothesis and conclude the mean active concentrations do differ significantly at $\alpha = 0.05$.

P-value $= 2\,(1-\Phi(2.385)) = 2(1-0.99146) = 0.0171$

The conclusions reached by the confidence interval of problem 5-10 and the test of hypothesis conducted here are the same. A two-sided confidence interval can be thought of as representing the “acceptance region” of a hypothesis test, given that the level of significance is the same for both procedures. Thus if the value δ falls outside the confidence interval, it is the same result as rejecting the null hypothesis.

5-14. Assume the populations follow normal distributions and $\sigma_1^2 = \sigma_2^2$. The assumption of equal variances may be permitted in this case since it is known that the t-test and confidence intervals involving the t-distribution are robust to this assumption of equal variances when sample sizes are equal.

Case 1: AFCC Case 2: ATC

μ_1 = mean foam expansion for AFCC
$\bar{x}_1 = 4.7$
$s_1 = 0.6$
$n_1 = 5$

μ_2 = mean foam expansion for ATC
$\bar{x}_2 = 6.9$
$s_2 = 0.8$
$n_2 = 5$

95% confidence interval: $t_{0.025,8} = 2.306$

$$s_p = \sqrt{\frac{5(0.60)+5(0.80)}{8}} = 0.7071$$

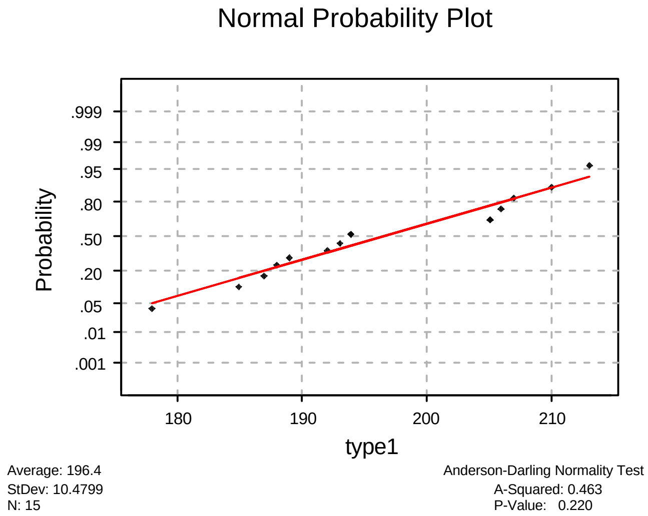
$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

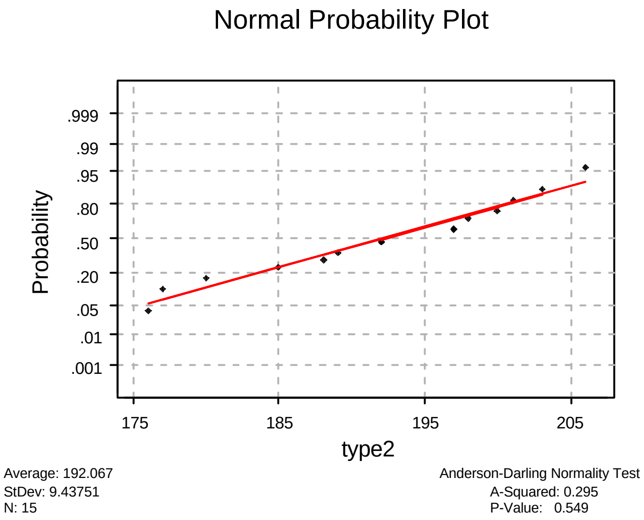
$$(4.7-6.9) - 2.306\sqrt{\frac{1}{5}+\frac{1}{5}} \le \mu_1 - \mu_2 \le \left(4.7-6.9\right) + 2.306\sqrt{\frac{1}{5}+\frac{1}{5}}$$

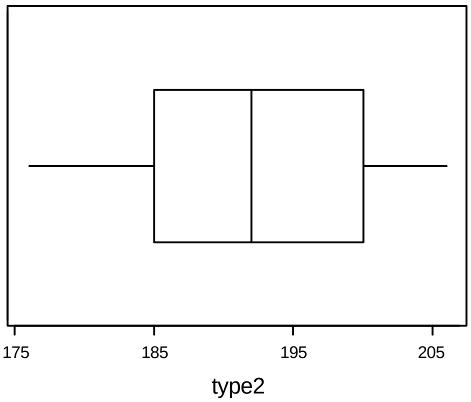
$$-3.23 \le \mu_1 - \mu_2 \le -1.17$$

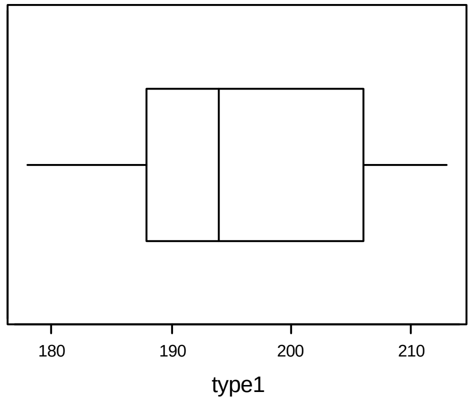
Yes, with 95% confidence, we believe the mean foam expansion for ATC exceeds that of AFCC by between 1.17 and 2.32.

5-16. a) According to the normal probability plots, the assumption of normality appears to be met since the data fall approximately along a straight line. The equality of variances does not appear to be severely violated either since the slopes are approximately the same for both samples.









b) 1) The parameter of interest is the difference in deflection temperature under load, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha, n_1+n_2-2}$ where $-t_{0.05,28} = -1.701$

7) Type 1 Type 2

$\bar{x}_1 = 196.4$ $\bar{x}_2 = 192.067$ $\Delta_0 = 0$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$s_1 = 10.48$ $s_2 = 9.44$

$$= \sqrt{\frac{14(10.48)^2 + 14(9.44)^2}{28}} = 9.97$$

$n_1 = 15$ $n_2 = 15$

$$t_0 = \frac{(196.4 - 192.067) - 0}{9.97\sqrt{\frac{1}{15} + \frac{1}{15}}} = 1.19$$

8) Since $1.19 > -1.701$ do not reject the null hypothesis and conclude the mean deflection temperature under load for type 2 does not significantly exceed the mean deflection temperature under load for type 1 at the 0.05 level of significance.

c) P-value = $2P(t > 1.19)$ $0.75 < \text{p-value} < 0.90$

d) $\Delta = 5$ Use s_p as an estimate of σ:

$$d = \frac{\mu_2 - \mu_1}{2s_p} = \frac{5}{2(9.97)} = 0.251$$

Using Chart V g) with $\beta = 0.10$, d = 0.251 we get $n \cong 100$. So, $n_1 = n_2 = 100$; Therefore, the sample sizes of 15 are inadequate.

5-18. a) 1) The parameter of interest is the difference in mean impact strength, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha,\nu}$ where $t_{0.05,25} = -1.708$ since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1+1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2+1}} - 2 = 27.43 - 2 = 25.43$$

$\nu \cong 25$

(truncated)

7) $\bar{x}_1 = 290$ $\bar{x}_2 = 321$

$s_1 = 12 \qquad s_2 = 22$

$n_1 = 10 \qquad n_2 = 16$

$$t_0 = \frac{(290 - 321) - 0}{\sqrt{\frac{(12)^2}{10} + \frac{(22)^2}{16}}} = -4.64$$

8) Since –4.64 < –1.708 reject the null hypothesis and conclude that supplier 2 provides gears with higher mean impact strength at the 0.05 level of significance.

b) P-value = P(t < –4.64): P-value < 0.0005

c) 1) The parameter of interest is the difference in mean impact strength, $\mu_2 - \mu_1$

2) $H_0: \mu_2 - \mu_1 = 25$

3) $H_1: \mu_2 - \mu_1 > 25$ or $\mu_2 > \mu_1 + 25$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_2 - \bar{x}_1) - \delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 > t_{\alpha,\nu} = 1.708$ where

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 + 1} + \frac{\left(\frac{s_2^2}{n_2}\right)}{n_2 + 1}} - 2 = 27.43 - 2 = 25.43$$

$\nu \cong 25$

7) $\bar{x}_1 = 290 \qquad \bar{x}_2 = 321 \qquad \Delta_0 = 25$

$s_1 = 12 \qquad s_2 = 22$

$n_1 = 10 \qquad n_2 = 16$

$$t_0 = \frac{(321 - 290) - 25}{\sqrt{\frac{(12)^2}{10} + \frac{(22)^2}{16}}} = 0.898$$

8) Since 0.898 < 1.708, do not reject the null hypothesis and conclude that the mean impact strength from supplier 2 is not at least 25 ft-lb higher that supplier 1 using α = 0.05.

5-20. 1) The parameter of interest is the difference in mean melting point, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.02$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.01,40} = -2.423$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where $t_{0.01,40} = 2.423$

7) $\bar{x}_1 = 421 \qquad \bar{x}_2 = 426 \qquad \Delta_0 = 0 \qquad s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$

$s_1 = 4 \qquad s_2 = 3$

$$= \sqrt{\frac{20(4)^2 + 20(3)^2}{40}} = 2.915$$

$n_1 = 21 \qquad n_2 = 21$

$$t_0 = \frac{(421 - 426) - 0}{2.915\sqrt{\frac{1}{20} + \frac{1}{20}}} = -5.424$$

8) Since $-5.424 < -2.423$ reject the null hypothesis and conclude that the data do not support the claim that both alloys have the same melting point at $\alpha = 0.02$

P-value = $2P\left(t < -5.424\right)$ P-value < 0.0010

5-22. a) 1) The parameter of interest is the difference in mean wear amount, $\mu_1 - \mu_2$.

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{0.025,27}$ where $-t_{0.025,27} = -2.052$ or $t_0 > t_{0.025,27}$ where

$t_{0.025,27} =$ 2.052 since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 + 1} + \frac{\left(\frac{s_2^2}{n_2}\right)}{n_2 + 1}} - 2 = 29.24 - 2 = 27.43$$

$\nu \cong 27$

(truncated)

7) $\overline{x}_1 = 20 \quad \overline{x}_2 = 15 \qquad \Delta_0 = 0$

$s_1 = 2 \qquad s_2 = 8$

$n_1 = 25 \qquad n_2 = 25$

$$t_0 = \frac{(20 - 15) - 0}{\sqrt{\frac{(2)^2}{25} + \frac{(8)^2}{25}}} = 3.03$$

8) Since 3.03 > 2.052 reject the null hypothesis and conclude that the data support the claim that the two companies produce material with significantly different wear at the 0.05 level of significance.

b) P-value = 2P(t > 3.03), 2(0.0025) < P-value < 2(0.005)

0.005 < P-value < 0.010

c) 1) The parameter of interest is the difference in mean wear amount, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$

3) $H_1: \mu_1 - \mu_2 > 0$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

6) Reject the null hypothesis if $t_0 > t_{0.05,27}$ where $t_{0.05,27} = 1.703$ since

$$\nu = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(\dfrac{s_1^2}{n_1}\right)^2}{n_1+1} + \dfrac{\left(\dfrac{s_2^2}{n_2}\right)}{n_2+1}} - 2 = 29.24 - 2 = 27.43$$

$$\nu \cong 27$$

7) $\bar{x}_1 = 20 \quad \bar{x}_2 = 15$

$s_1 = 2 \quad s_2 = 8 \qquad \Delta_0 = 0$

$n_1 = 25 \quad n_2 = 25$

$$t_0 = \frac{(20-15)-0}{\sqrt{\dfrac{(2)^2}{25} + \dfrac{(8)^2}{25}}} = 3.03$$

8) Since 3.03 > 1.703 reject the null hypothesis and conclude that the data support the claim that the material from company 1 has a higher mean wear than the material from company 2 using a 0.05 level of significance.

5-24. If $\alpha = 0.01$, construct a 99%lower one-sided confidence interval on the difference to answer this question. $t_{0.01,19} = 2.539$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2$$

$$(103.5 - 99.7) - 2.539\sqrt{\frac{(10.2)^2}{12} + \frac{(20.1)^2}{13}} \le \mu_1 - \mu_2$$

$$-12.21 \le \mu_1 - \mu_2 \,.$$

Since the interval contains 0, we are 99% confident there is no difference in the mean coating thickness between the two temperatures; that is, raising the process temperature does not significantly reduce the mean coating thickness.

5-26. $\bar{d} = 0.2736 \quad s_d = 0.1356$, $n = 9$

95% confidence interval:

$$\bar{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$0.2736 - 2.306\left(\frac{0.1356}{\sqrt{9}}\right) \le \mu_d \le 0.2736 + 2.306\left(\frac{0.1356}{\sqrt{9}}\right)$$

$$0.1694 \le \mu_d \le 0.3778$$

With 95% confidence, we believe the mean shear strength of Karlsruhe method exceeds the mean shear strength of the Lehigh method by between 0.1714 and 0.3758. Since 0 is not included in this interval, the interval is consistent with rejecting the null hypothesis that the means are the same.

The 95% confidence interval is directly related to a test of hypothesis with 0.05 level of significance, and the conclusions reached are identical.

5-28. 1) The parameter of interest is the difference between the mean parking times, $\mu_{d.}$

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d \neq 0$

4) $\alpha = 0.10$

5) The test statistic is

$$t_0 = \frac{\overline{d}}{s_d / \sqrt{n}}$$

6) Reject the null hpothesis if $t_0 < -t_{0.05,13}$ where $-t_{0.05,13} = -1.771$ or $t_0 > t_{0.05,13}$ where $t_{0.05,13} = 1.771$

7) $\overline{d} = 1.21$
$s_d = 12.68$
$n = 14$

$$t_0 = \frac{1.21}{12.68 / \sqrt{14}} = 0.357$$

8) Since $-1.771 < 0.357 < 1.771$ do not reject the null and conclude the data do not support the claim that the two cars have different mean parking times at the 0.10 level of significance. The result is consistent with the confidence interval constructed since 0 is included in the 90% confidence interval.

5-30. $\overline{d} = 868.375$ $s_d = 1290$, $n = 8$ where d_i = brand 1 - brand 2

99% confidence interval:

$$\overline{d} - t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d \le \overline{d} + t_{\alpha/2,n-1}\left(\frac{s_d}{\sqrt{n}}\right)$$

$$868.375 - 3.499\left(\frac{1290}{\sqrt{8}}\right) \le \mu_d \le 868.375 + 3.499\left(\frac{1290}{\sqrt{8}}\right)$$

$$-727.46 \le \mu_d \le 2464.21$$

Since zero is contained within this interval, we are 99% confident there is no significant difference between the two brands of tire.

5-32. 1) The parameter of interest is the difference in blood cholesterol level, μ_d where d_i = Before – After.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d > 0$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\overline{d}}{s_d / \sqrt{n}}$$

6) Reject the null hpothesis if $t_0 > t_{0.05,14}$ where $t_{0.05,14} = 1.761$

7) $\overline{d} = 26.867$
$s_d = 19.04$
$n = 15$

$$t_0 = \frac{26.867}{19.04 / \sqrt{15}} = 5.465$$

8) Since $5.465 > 1.761$ reject the null and conclude the data support the claim that low the mean difference in cholesterol levels is significanlty less after fat diet and aerobic exercise program at the 0.05 level of significance.

5-34. 1) The parameter of interest is the difference in mean weight, μ_d where d_i =Weight Before – Weight After.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d > 0$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

6) Reject the null hpothesis if $t_0 > t_{0.05,9}$ where $t_{0.05,9} = 1.833$

7) $\bar{d} = 17$
$s_d = 6.41$
$n = 10$

$$t_0 = \frac{17}{6.41/\sqrt{10}} = 8.387$$

8) Since 8.387 > 1.833 reject the null and conclude there is evidence to conclude that the mean weight loss is significantly greater than 0; that is, the data support the claim that this particular diet modification program is significantly effective in reducing weight at the 0.05 level of significance.

5-36. 1) The parameter of interest is the difference in mean weight loss, μ_d where d_i = Before − After.

2) $H_0 : \mu_d = 10$

3) $H_1 : \mu_d > 10$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\bar{d} - \Delta_0}{s_d / \sqrt{n}}$$

6) Reject the null hpothesis if $t_0 > t_{0.05,9}$ where $t_{0.05,9} = 1.833$

7) $\bar{d} = 17$
$s_d = 6.41$
$n = 10$

$$t_0 = \frac{17 - 10}{6.41/\sqrt{10}} = 3.45$$

8) Since 3.45 > 1.833 reject the null and conclude there is evidence to support the claim that this particular diet modification program is effective in producing a mean weight loss of at least 10 lbs at the 0.05 level of significance.

5-38. a) $f_{0.25,5,10} = 1.59$ d) $f_{0.75,5,10} = \frac{1}{f_{0.25,10,5}} = \frac{1}{1.89} = 0.529$

b) $f_{0.10,24,9} = 2.28$ e) $f_{0.90,24,9} = \frac{1}{f_{0.10,9,24}} = \frac{1}{1.91} = 0.524$

c) $f_{0.05,8,15} = 2.64$ f) $f_{0.95,8,15} = \frac{1}{f_{0.05,15,8}} = \frac{1}{3.22} = 0.311$

5-40. 1) The parameters of interest are the variances of concentration, σ_1^2, σ_2^2

2) $H_0 : \sigma_1^2 = \sigma_2^2$

3) $H_1 : \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.975,9,15}$ where $f_{0.975,9,15} = 0.265$ or $f_0 > f_{0.025,9,15}$ where $f_{0.025,9,15} = 3.12$

7) $n_1 = 10$ $n_2 = 16$

$s_1 = 4.7 \qquad s_2 = 5.8$

$$f_0 = \frac{(4.7)^2}{(5.8)^2} = 0.657$$

8) Since 0.265 < 0.657 < 3.12 do not reject the null hypothesis and conclude there is insufficient evidence to indicate the two population variances differ significantly at the 0.05 level of significance.

5-42. a) 90% confidence interval for the ratio of variances:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2, n_1-1, n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2, n_1-1, n_2-1}$$

$$\left(\frac{(0.35)^2}{(0.40)^2}\right) 0.412 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.35)^2}{(0.40)^2}\right) 2.33$$

$$0.3605 \le \frac{\sigma_1^2}{\sigma_2^2} \le 2.039$$

Since the interval contains 1, we are 90% confident the variances for the rod diameters are not significantly different.

b) 95% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2, n_1-1, n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2, n_1-1, n_2-1}$$

$$\left(\frac{(0.35)^2}{(0.40)^2}\right) 0.345 \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{(0.35)^2}{(0.40)^2}\right) 2.75$$

$$0.302 \le \frac{\sigma_1^2}{\sigma_2^2} \le 2.406$$

We are 95% confident the variances for the rod diameters are not significantly different.

The 95% confidence interval is wider than the 90% confidence interval.

c) 90% lower-sided confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha, n_1-1, n_2-1} \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$\left(\frac{(0.35)^2}{(0.40)^2}\right) 0.503 \le \frac{\sigma_1^2}{\sigma_2^2}$$

$$0.440 \le \frac{\sigma_1^2}{\sigma_2^2}$$

With 90% confidence, we believe the variances for the rod diameters are not significantly different from each other.

5-44. 1) The parameters of interest are the thickness variances, σ_1^2, σ_2^2

2) $H_0 : \sigma_1^2 = \sigma_2^2$

3) $H_1 : \sigma_1^2 \ne \sigma_2^2$

4) $\alpha = 0.02$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.99,7,7}$ where $f_{0.99,7,7} = 0.143$ or $f_0 > f_{0.01,7,7}$ where $f_{0.01,7,7} = 6.99$

7) $n_1 = 8 \qquad n_2 = 8$

$s_1 = 0.11 \qquad s_2 = 0.09$

$$f_0 = \frac{(0.11)^2}{(0.09)^2} = 1.49$$

8) Since 0.143 < 1.232 < 6.99 do not reject the null hypothesis and conclude the thickness variances do not significantly differ at the 0.02 level of significance.

5-46. 1) The parameters of interest are the melting variances, σ_1^2, σ_2^2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{0.975,20,20}$ where $f_{0.975,20,20} = 0.4065$ or $f_0 > f_{0.025,20,20}$ where $f_{0.025,20,20} = 2.46$

7) $n_1 = 21 \qquad n_2 = 21$

$s_1 = 4 \qquad s_2 = 3$

$$f_0 = \frac{(4)^2}{(3)^2} = 1.78$$

8) Since 0.4065 < 1.78 < 2.46 do not reject the null hypothesis and conclude the population variances do not significantly differ at the 0.05 level of significance.

5-48. 1) The parameters of interest are the time to assemble standard deviations, σ_1, σ_2

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) $\alpha = 0.02$

5) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

6) Reject the null hypothesis if $f_0 < f_{1-\alpha/2,n_1-1,n_2-1} = 0..365$ or $f_0 > f_{\alpha/2,n_1-1,n_2-1} = 2.86$

7) $n_1 = 25 \qquad n_2 = 21$

$s_1 = 0.98 \qquad s_2 = 1.02$

$$f_0 = \frac{(1.02)^2}{(0.98)^2} = 0.923$$

8) Since 0.365 < 0.923 < 2.86 do not reject the null hypothesis and conclude there is no evidence to support the claim that men and women differ significantly in repeatability for this assembly task at the 0.02 level of significance.

5-50. 1) the parameters of interest are the proportion of defective parts, p_1 and p_2

2) $H_0: p_1 = p_2$

3) $H_1: p_1 \neq p_2$

4) $\alpha = 0.05$

5) Test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where}$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

6) Reject the null hypothesis if $z_0 < -z_{0.025}$ where $-z_{0.025} = -1.96$ or $z_0 > z_{0.025}$ where $z_{0.025} = 1.96$

7) $n_1 = 300$ $\quad n_2 = 300$

$x_1 = 15$ $\quad x_2 = 8$

$\hat{p}_1 = 0.05$ $\quad \hat{p}_2 = 0.0267$ $\quad \hat{p} = \dfrac{15+8}{300+300} = 0.0383$

$$z_0 = \frac{0.05 - 0.0267}{\sqrt{0.0383(1-0.0383)\left(\frac{1}{300}+\frac{1}{300}\right)}} = 1.49$$

8) Since $-1.96 < 1.49 < 1.96$ do not reject the null hypothesis and conclude that yes the evidence indicates that there is not a significant difference in the fraction of defective parts produced by the two machines at the 0.05 level of significance.

P-value = $2(1-P(z < 1.49)) = 0.13622$

5-52. a) $$\beta = \Phi\left(\frac{z_{\alpha/2}\sqrt{\overline{pq}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} - (p_1 - p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right) - \Phi\left(\frac{-z_{\alpha/2}\sqrt{\overline{pq}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} - (p_1 - p_2)}{\hat{\sigma}_{\hat{p}_1-\hat{p}_2}}\right)$$

$\overline{p} = \dfrac{300(0.05)+300(0.02)}{300+300} = 0.035$ $\qquad \overline{q} = 0.965$

$$\hat{\sigma}_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{0.05(1-0.05)}{300}+\frac{0.02(1-0.02)}{300}} = 0.015$$

$\beta =$

$$\Phi\left(\frac{1.96\sqrt{0.035(0.965)\left(\frac{1}{300}+\frac{1}{300}\right)} - (0.05-0.02)}{0.015}\right) - \Phi\left(\frac{-1.96\sqrt{0.035(0.965)\left(\frac{1}{300}+\frac{1}{300}\right)} - (0.05-0.02)}{0.015}\right)$$

$$= \Phi(-0.04) - \Phi(-3.96) = 0.48405 - 0.00004 = 0.48401$$

Power = $1 - 0.48401 = 0.51599$

b) $$n = \frac{\left(z_{\alpha/2}\sqrt{\frac{(p_1+p_2)(q_1+q_2)}{2}} + z_\beta\sqrt{p_1q_1+p_2q_2}\right)^2}{(p_1-p_2)^2}$$

$$= \frac{\left(1.96\sqrt{\frac{(0.05+0.02)(0.95+0.98)}{2}} + 1.29\sqrt{0.05(0.95)+0.02(0.98)}\right)^2}{(0.05-0.02)^2} = 790.67$$

n = 791

5-54. 95% confidence interval on the difference:

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \le p_1 - p_2 \le (\hat{p}_1 - \hat{p}_2) + z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$(0.05-0.0267)-1.96\sqrt{\frac{0.05(1-0.05)}{300}+\frac{0.0267(1-0.0267)}{300}} \le p_1-p_2 \le (0.05-0.0267)+1.96\sqrt{\frac{0.05(1-0.05)}{300}+\frac{0.0267(1-0.0267)}{300}}$$

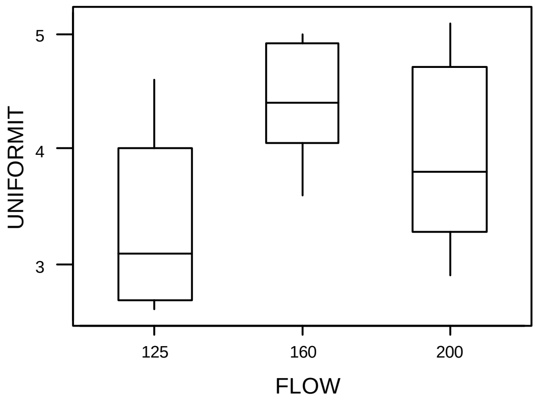
$-0.0074 \le p_1 - p_2 \le 0.054$

Since this interval contains the value zero, we are 95% confident there is no significant difference in the fraction of defective parts produced by the two machines and that the difference in proportions is between –0.0074 and 0.054.

5-56. a)

```
                                One-Way Analysis of Variance
----------------------------------------------------------------------------
Data: Observation

Level codes: SCCM

Labels:

Means plot: LSD           Confidence level: 90       Range test: LSD


                           Analysis of variance
------------------------------------------------------------------------------
Source of variation   Sum of Squares   d.f.   Mean square    F-ratio  Sig.
level
------------------------------------------------------------------------------
Between groups             3.6477778      2     1.8238889      3.586     .0534
Within groups              7.6300000     15      .5086667
------------------------------------------------------------------------------
Total (corrected)          11.277778     17

0 missing value(s) have been excluded.
```

Reject H_0 at $\alpha = 0.01$. C_2F_6 flow rate does appear to affect etch uniformity.



b) Examining the box plots, the 125 and 160 mean levels seem to be the most different.

5-58. a)

```
                          One-Way Analysis of Variance
------------------------------------------------------------------------------
Data: Density

Level codes: Temp

Labels:
```

```
Means plot: LSD            Confidence level: 95       Range test: LSD


                             Analysis of variance
------------------------------------------------------------------------------
Source of variation   Sum of Squares   d.f.   Mean square    F-ratio  Sig.
level
------------------------------------------------------------------------------
Between groups              .1391104      3      .0463701      2.616     .0827
Within groups               .3190714     18      .0177262
------------------------------------------------------------------------------
Total (corrected)           .4581818     21

0 missing value(s) have been excluded.
```

Do not reject H_0. There is insignificant evidence to indicate the four firing temperatures affect the density of the brick.

b) P-value = 0.0827

5-60.

```
                         One-Way Analysis of Variance
------------------------------------------------------------------------------
Data: Conductivity

Level codes: Coating

Labels:

Means plot: LSD            Confidence level: 95       Range test: LSD


                             Analysis of variance
------------------------------------------------------------------------------
Source of variation   Sum of Squares   d.f.   Mean square    F-ratio  Sig.
level
------------------------------------------------------------------------------
Between groups             1060.5000      4     265.12500     16.349     .0000
Within groups               243.2500     15      16.21667
------------------------------------------------------------------------------
Total (corrected)          1303.7500     19

0 missing value(s) have been excluded.
```

Reject H_0. There appears to be a significant difference among the five coating types in their effect on conductivity.

Chapter 6 (Section 6.1-6.4)

6-2. a) $y_0 = \beta_0 + \beta_1 x_1$

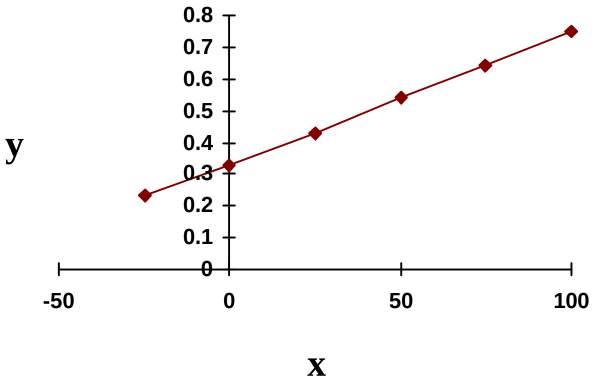
$$S_{xx} = 143215.8 - \frac{1478^2}{20} = 33991.6$$

$$S_{xy} = 1083.67 - \frac{(1478)(12.75)}{20} = 141.445$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{141.445}{33991.6} = 0.0041612$$

$$\hat{\beta}_0 = \frac{12.75}{20} - (0.0041612)(\frac{1478}{20}) = 0.3299892$$

$$\hat{y} = 0.3299892 + 0.0041612x$$



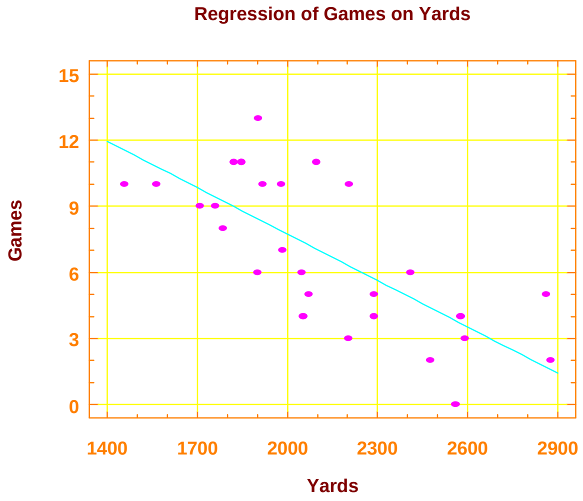
b) $\hat{y} = 0.3299892 + 0.0041612(85) = 0.683689$

c) $\hat{y} = 0.3299892 + 0.0041612(90) = 0.7044949$

d) $\hat{\beta}_1 = 0.00416$

6-4. a)

```
Regression Analysis - Linear model: Y = a+bX
------------------------------------------------------------------------------
Dependent variable: Games                    Independent variable: Yards
------------------------------------------------------------------------------
                                 Standard              T              Prob.
Parameter       Estimate           Error             Value            Level
------------------------------------------------------------------------------
Intercept        21.7883          2.69623            8.081           .00000
Slope         -7.0251E-3       1.25965E-3         -5.57703           .00001
------------------------------------------------------------------------------
                           Analysis of Variance
------------------------------------------------------------------------------
Source                 Sum of Squares     Df   Mean Square    F-Ratio  Prob. Level
Model                       178.09231      1     178.09231    31.1032       .00001
Residual                    148.87197     26       5.72585
------------------------------------------------------------------------------
Total (Corr.)               326.96429     27
Correlation Coefficient = -0.738027           R-squared =   54.47 percent
Stnd. Error of Est. = 2.39287
```



If the calculations were to be done by hand use Equations (6-13) and (6-14).

b) $\hat{y} = 21.7883 - 0.0070251(1800) = 9.143$

c) $-0.0070251(-100) = 0.70251$ games won.

d) $\frac{1}{0.0070251} = 142.35$ yds decrease required.

e) $\hat{y} = 21.7883 - 0.0070251(1917) = 8.321$

$$e_i = y_i - \hat{y}_i$$
$$= 10 - 8.321 = 1.679$$

6-6. a)

```
Regression Analysis - Linear model: Y = a+bX
------------------------------------------------------------------------------
Dependent variable: Usage                     Independent variable: Temperature
------------------------------------------------------------------------------
                                  Standard               T              Prob.
Parameter        Estimate           Error              Value             Level
------------------------------------------------------------------------------
Intercept         -6.3355         1.66765           -3.79906             .00349
Slope             9.20836       0.0337744            272.643             .00000
------------------------------------------------------------------------------
                           Analysis of Variance
------------------------------------------------------------------------------
Source                 Sum of Squares     Df   Mean Square     F-Ratio  Prob. Level
Model                       280583.12      1     280583.12     74334.4       .00000
Residual                    37.746089     10      3.774609
------------------------------------------------------------------------------
Total (Corr.)               280620.87     11
Correlation Coefficient = 0.999933           R-squared =   99.99 percent
Stnd. Error of Est. = 1.94284
```

If the calculations were to be done by hand use Equations (6-13) and (6-14).
$\hat{y} = -6.3355 + 9.20836x$

b) $\hat{y} = -6.3355 + 9.20836(55) = 500.124$

c) If monthly temperature increases by $1^{\circ}F$, $\hat{y}$ increases by 9.20836.

d) $\hat{y} = -6.3355 + 9.20836(47) = 426.458$
$\hat{y} = 426.458$
$e = y - \hat{y} = 424.84 - 426.458 = -1.618$

6-8.

```
                       Model fitting results for:  y
------------------------------------------------------------------------------
Independent variable             coefficient  std. error     t-value
sig.level
------------------------------------------------------------------------------
CONSTANT                          350.994271   74.753074      4.6954
0.0183
x1                                 -1.271994     1.16914     -1.0880
0.3562
x2                                 -0.153904     0.08953     -1.7190
0.1841
------------------------------------------------------------------------------
R-SQ. (ADJ.) = 0.7696  SE=      25.497858  MAE=      16.319125  DurbWat=
2.565
Previously:    0.0000            0.000000             0.000000
0.000
6 observations fitted, forecast(s) computed for 0 missing val. of dep. var.
```

If the calculations were to be done by hand use Equation (6-21).

a) $\hat{y} = 350.9943 - 1.272x_1 - 0.1539x_2$

b) $\hat{y} = 350.9943 - 1.272(25) - 0.1539(1000) = 165.29$

c)

```
                       Model fitting results for: y
------------------------------------------------------------------------------
Independent variable             coefficient  std. error     t-value
sig.level
------------------------------------------------------------------------------
CONSTANT                          125.865548  197.957166      0.6358
0.5899
x1                                  7.758641    7.514795      1.0324
0.4104
x2                                  0.094304    0.220657      0.4274
0.7107
x1*x2                               0.009186    0.007564     -1.2145
0.3485
------------------------------------------------------------------------------
R-SQ. (ADJ.) = 0.8011  SE=      23.691404  MAE=      12.527757  DurbWat=
3.003
Previously:    0.7696           25.497858            16.319125
2.565
6 observations fitted, forecast(s) computed for 0 missing val. of dep. var.
```

If the calculations were to be done by hand, add a cross-product column to the **X** matrix and use Equation (6-21).

$\hat{y}' = 125.8655 - 7.7586x_1 - 0.0943x_2 - 0.0092x_1x_2$

d) $\hat{y}' = 125.8655 - 7.7586(25) - 0.0943(1000) - 0.0092(25)(1000)$
$\hat{y}' = 184.13$ The predicted value is larger

6-10.

```
                       Model fitting results for: y
```

```
------------------------------------------------------------------------------
Independent variable            coefficient  std. error     t-value
sig.level
------------------------------------------------------------------------------
CONSTANT                          47.173999   49.581476      0.9514
0.3555
x1                                -9.735202    3.691625     -2.6371
0.0179
x2                                 0.428287    0.223933      1.9126
0.0739
x3                                18.237455    1.311802     13.9026
0.0000
------------------------------------------------------------------------------
R-SQ. (ADJ.) = 0.9925  SE=       3.479627  MAE=       2.511105  DurbWat=
1.778
Previously:    0.0000            0.000000             0.000000
0.000
20 observations fitted, forecast(s) computed for 0 missing val. of dep. var.
```

If the calculations were to be done by hand, add a cross-product column to the **X** matrix and use Equation (6-21).

a) $\hat{y} = 47.174 - 9.7352x_1 + 0.4283x_2 + 18.2375x_3$

b) $\hat{y} = 47.174 - 9.7352(14.5) + 0.4283(220) + 18.2375(5) = 91.43$

6-12. a) 1) The parameter of interest is the regressor variable coefficient, β_1

2) $H_0{:}\beta_1 = 0$

3) $H_1{:}\beta_1 \neq 0$

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{MS_R}{MS_E} = \frac{SS_R / 1}{SS_E / (n-2)}$$

6) Reject H_0 if $f_0 > f_{\alpha,1,12}$ where $f_{0.05,1,12} = 4.75$

7) Using results from Exercise 6-1

$$\begin{aligned} SS_R &= \hat{\beta}_1 S_{xy} = -2.3298017(-59.057143) \\ &= 137.59 \\ SS_E &= S_{yy} - SS_R \\ &= 159.71429 - 137.59143 \\ &= 22.123 \end{aligned}$$

$$f_0 = \frac{137.59}{22.123/12} = 74.63$$

8) Since 74.63 > 4.75 reject H_0 and conclude that compressive strength is a significant in predicting intrinsic permeability of concrete at $\alpha = 0.05$. We can therefore conclude model specifies a useful linear relationship between these two variables.

$P - \text{value} \cong 0.000002$

b) $\hat{\sigma}^2 = MS_E = \frac{SS_E}{n-2} = \frac{22.123}{12} = 1.844$

6-14. a) Refer to ANOVA table of Exercise 6-4.

1) The parameter of interest is the regressor variable coefficient, β_1.

2) $H_0{:}\beta_1 = 0$

3) $H_1{:}\beta_1 \neq 0$

4) $\alpha = 0.01$

5) The test statistic is

$$f_0 = \frac{MS_R}{MS_E} = \frac{SS_R / 1}{SS_E / (n-2)}$$

6) Reject H_0 if $f_0 > f_{\alpha,1,26}$ where $f_{0.01,1,26} = 7.724$

7) Using the results of Exercise 6-4

$$f_0 = \frac{MS_R}{MS_E} = 31.1032$$

8) Since 31.1032 > 7.724 reject H_0 and conclude the model is useful at $\alpha = 0.01$.

$P - \text{value} = 0.000007$

b) $\hat{\sigma}^2 = MS_E = 5.72585$

c) 2) $H_0{:}\beta_1 = 0$

3) $H_1{:}\beta_1 \neq 0$

4) $\alpha = 0.01$

5) The test statistic is

$$t_0 = \frac{\hat{\beta}_1}{se(\hat{\beta}_1)}$$

6) Reject H_0 if $t_0 < -t_{\alpha/2,n-2}$ where $-t_{0.005,26} = -2.78$ or $t_0 > t_{0.005,26} = 2.78$

7) Using the results from Exercise 6-4

$$t_0 = \frac{-0.0070251}{0.00125965} = -5.577$$

8) Since $-5.577 < -2.78$ reject H_0 and conclude the regressor is useful in the model at $\alpha = 0.01$.

6-16. Refer to ANOVA for Exercise 6-6

a) 1) The parameter of interest is the regressor variable coefficient, β_1.

2) $H_0{:}\beta_1 = 0$

3) $H_1{:}\beta_1 \neq 0$

4) $\alpha = 0.01$

5) The test statistic is

$$f_0 = \frac{MS_R}{MS_E} = \frac{SS_R / 1}{SS_E / (n-2)}$$

6) Reject H_0 if $f_0 > f_{\alpha,1,22}$ where $f_{0.01,1,10} = 10.049$

7) Using the results from Exercise 6-6

$$f_0 = \frac{280583.12 / 1}{37.746089 / 10} = 74334.4$$

8) Since 74334.4 > 10.049, reject H_0 and conclude the model is useful $\alpha = 0.01$.

P-value < 0.000001

b) MSE = $SS_E/(n-p) = \hat{\sigma}^2 = 3.77461$

c) 1) The parameter of interest is the regressor variable coefficient, β_1.
2) $H_0: \beta_1 = 10$
3) $H_1: \beta_1 \neq 10$
4) $\alpha = 0.01$
5) The test statistic is

$$t_0 = \frac{\hat{\beta}_1 - \beta_{1,0}}{se(\hat{\beta}_1)}$$

6) Reject H_0 if $t_0 < -t_{\alpha/2,n-2}$ where $-t_{0.005,10} = -3.17$ or $t_0 > t_{0.005,10} = 3.17$
7) Using the results from Exercise 6-6

$$t_0 = \frac{9.21 - 10}{0.0338} = -23.37$$

8) Since $-23.37 < -3.17$ reject H_0 and conclude the regressor is useful at $\alpha = 0.01$.
P-value = 0.

6-18. a) $\beta_1: t_0 = 4.82$ P-value = 2(4.08 E-5) = 8.16 E-5
$\beta_2: t_0 = 8.21$ P-value = 2(1.91 E-8) = 3.82 E-8
$\beta_3: t_0 = 0.98$ P-value = 2 (0.1689) = 0.3378

b) 2) $H_0: \beta_3 = 0$
3) $H_1: \beta_3 \neq 0$
4) $\alpha = 0.05$
5) Reject H_0 if $t_0 < -t_{\alpha/2,n-2}$ where $-t_{0.025,23} = -2.074$ or $t_0 > t_{\alpha/2,n-2}$ where $t_{0.025,23} = 2.074$
6) $t_0 = 0.98$
7) Since $-2.074 < 0.98 < 2.074$ do not reject H_0 and conclude that x_3 is not useful as a regressor in the model at $\alpha =$ 0.05.

6-20.

```
                 Analysis of Variance for the Full Regression
-----------------------------------------------------------------------------
Source              Sum of Squares     DF    Mean Square      F-Ratio   P-
value
-----------------------------------------------------------------------------
Model                      12161.6      2        6080.79      9.35303
.0514
Error                      1950.42      3        650.141
-----------------------------------------------------------------------------
Total (Corr.)              14112.0      5

R-squared = 0.86179                                  Stnd. error of est. =
25.4979
R-squared (Adj. for d.f.) = 0.76965              Durbin-Watson statistic =
2.56509
```

Assume no interaction.

a) 1) The parameters of interest are the regression coefficients, β_j .
2) $H_0: \beta_1 = \beta_2 = 0$
3) H_1: at least one $\beta_j \neq 0$
4) $\alpha = 0.05$
5) The test statistic is

$$f_0 = \frac{SS_R / k}{SS_E / (n-p)}$$

6) Reject H_0 if $f_0 > f_{\alpha,2,3}$ where $f_{0.05,2,3} = 9.55$
7) Using the results from the ANOVA Table above

$$f_0 = 9.35$$

If f_0 is to be calculated by hand, use Equation (6-34).

8) Since 9.35 < 9.55 do not reject H_0 and conclude the regressors are not significantly useful as regressors in the
model at $\alpha = 0.05$.

P-value = 0.0514

b)

```
                      Model fitting results for: y
------------------------------------------------------------------------------
Independent variable             coefficient  std. error     t-value
sig.level
------------------------------------------------------------------------------
CONSTANT                          350.994271   74.753074      4.6954
0.0183
x1                                 -1.271994     1.16914     -1.0880
0.3562
x2                                 -0.153904     0.08953     -1.7190
0.1841
------------------------------------------------------------------------------
R-SQ. (ADJ.) = 0.7696  SE=      25.497858  MAE=      16.319125  DurbWat=
2.565
Previously:    0.0000            0.000000             0.000000
0.000
6 observations fitted, forecast(s) computed for 0 missing val. of dep. var.
```

$H_0{:}\beta_1 = 0$	$H_0{:}\beta_2 = 0$
$H_1{:}\beta_1 \neq 0$	$H_1{:}\beta_2 \neq 0$
$t_0 = -1.09$	$t_0 = -1.72$
$t_{\alpha/2,3} = t_{.025,3} = 3.182$	$t_{\alpha/2,3} = t_{.025,3} = 3.182$

Since $-3.182 < -1.09 < 3.182$ for β_1, and $-3.182 < -1.72 < 3.182$ for β_2, do not reject H_0 for either regressor and
conclude that x_1 and x_2 are not significantly useful as regressors for this model at $\alpha = 0.05$

If t_0 is to be calculated by hand, use Equation (6-38).

6-22.

```
                 Analysis of Variance for the Full Regression
------------------------------------------------------------------------------
Source                Sum of Squares     DF    Mean Square      F-Ratio   P-
value
------------------------------------------------------------------------------
Model                        30531.5      3        10177.2      840.546
.0000
Error                        193.725     16        12.1078
------------------------------------------------------------------------------
Total (Corr.)                30725.2     19

R-squared = 0.993695                               Stnd. error of est. =
3.47963
R-squared (Adj. for d.f.) = 0.992513           Durbin-Watson statistic =
1.77758
```

a) 1) The parameters of interest are the regression coefficients, β_j.

2) $H_0{:}\beta_j = 0$ for all j

3) $H_1{:}\beta_j \neq 0$ for at least one j

4) $\alpha = 0.05$

5) The test statistic is

$$f_0 = \frac{SS_R / k}{SS_E / (n-p)}$$

6) Reject H_0 if $f_0 > f_{\alpha,3,16}$ where $f_{0.05,3,16} = 3.24$

7) Using the results from the ANOVA table

$$f_0 = 840.55$$

If f_0 is to be calculated by hand, use Equation (6-34).

8) Since 840.55 > 3.24 reject H_0 and conclude the regressors are not significantly useful as regressors in the model at $\alpha = 0.05$.

b) $\hat{\sigma}^2 = 12.1078$

c)

```
                       Model fitting results for: y
----------------------------------------------------------------------------
--
Independent variable                coefficient  std. error     t-value
sig.level
----------------------------------------------------------------------------
--
CONSTANT                              47.173999   49.581476      0.9514
0.3555
x1                                    -9.735202    3.691625     -2.6371
0.0179
x2                                     0.428287    0.223933      1.9126
0.0739
x3                                    18.237455    1.311802     13.9026
0.0000
----------------------------------------------------------------------------
--
R-SQ. (ADJ.) = 0.9925  SE=       3.479627  MAE=       2.511105  DurbWat=
1.778
Previously:    0.0000            0.000000            0.000000
0.000
20 observations fitted, forecast(s) computed for 0 missing val. of dep. var.
```

$\alpha = 0.05$	$t_{\alpha/2,n-p} = t_{.025,16} = 2.12$	
$H_0{:}\beta_1 = 0$	$\beta_2 = 0$	$\beta_3 = 0$
$H_1{:}\beta_1 \neq 0$	$\beta_2 \neq 0$	$\beta_3 \neq 0$
$t_0 = -2.637$	$t_0 = 1.91$	$t_0 = 13.90$
$-2.637 < -2.12$	$-2.12 < 1.91 < 2.12$	$13.9 > 2.12$
Reject H_0	Do not reject H_0	Reject H_0

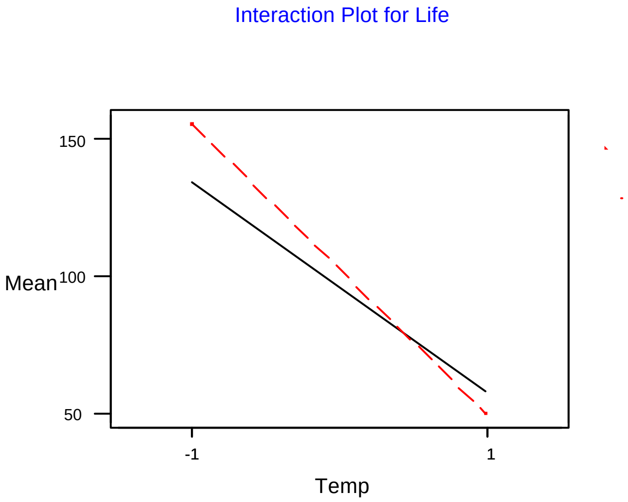
β_1 : Reject H_0 and conclude that x_1 is a significant regressor in the model at $\alpha = 0.05$.
β_2 : Do not reject H_0 and conclude that x_2 is not a significant regressor in the model at $\alpha = 0.05$.
β_3 : Reject H_0 and conclude that x_3 is a significant regressor in the model at $\alpha = 0.05$.

Chapter 7 (Problems 7.1-7.8)

7-1. a)

```
Estimated Effects and Coefficients for Life

Term             Effect      Coef  StDev Coef       T      P
Constant                    99.38       7.711   12.89  0.000
Material           6.50      3.25       7.711    0.42  0.681
Temp             -91.75    -45.87       7.711   -5.95  0.000
Material*Temp    -14.50     -7.25       7.711   -0.94  0.366
```

b)



The interaction plot does not indicate a strong interaction between temperature and material.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that temperature is significant, but material and the interaction material*temperature are not at the α = 0.05 level.

d) The 95% confidence intervals are given by effect estimate ± 2(s.e.(effect)) where s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Temperature: s.e.(effect) = 2(7.711) = 15.42
Approximate 95% confidence interval on the the effect of Temperature:

6.5 ± 2(15.42)
(–24.34, 37.34)

Material: s.e.(effect) = 2(7.711) = 15.42
Approximate 95% confidence interval for the effect of Material:

–91.75 ± 2(15.42)
(–122.59, –60.91)

Material*Temperature: s.e.(effect) = 2(7.711) = 15.42
Approximate 95% confidence interval for the effect of Material*Temperature:

–14.50 ± 2(15.42)
(–45.34, 16.34)

e)

```
The regression equation is
Life = 99.4 + 3.25 Material - 45.9 Temp - 7.25 Material*Temp
```

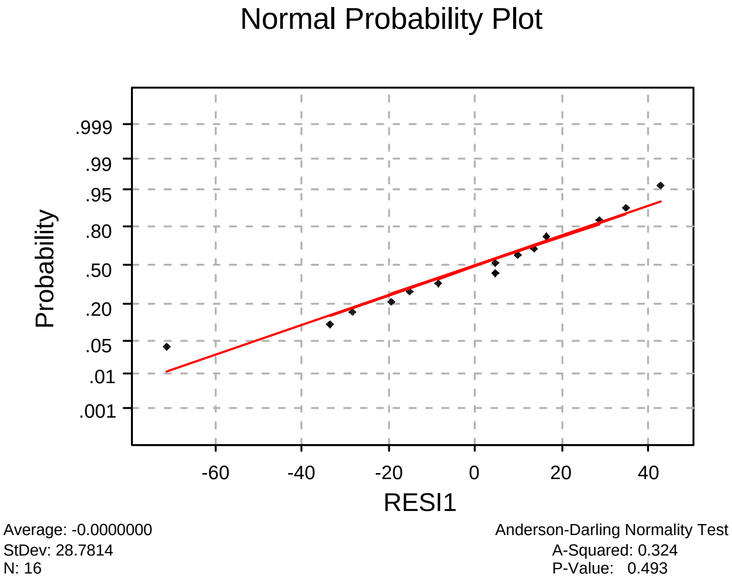
```
Predictor        Coef        StDev          T        P
Constant       99.375        7.711      12.89    0.000
Material        3.250        7.711       0.42    0.681
Temp          -45.875        7.711      -5.95    0.000
Material*Temp -7.250         7.711      -0.94    0.366
```

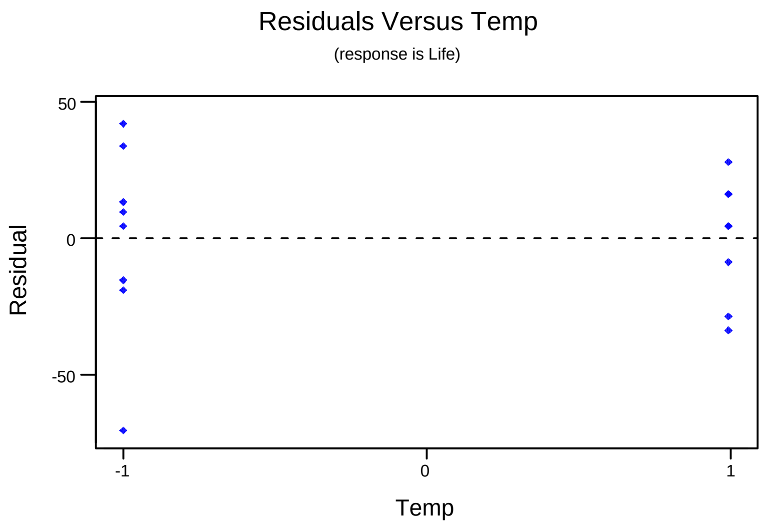
Based on the regression analysis, only temperature appears to be the significant factor. This result is equivalent to that obtained in part c.

The final regression analysis and model are

```
Life = 99.4 - 45.9 Temp

Predictor        Coef        StDev          T        P
Constant       99.375        7.448      13.34    0.000
Temp          -45.875        7.448      -6.16    0.000

S = 29.79        R-Sq = 73.0%     R-Sq(adj) = 71.1%

Analysis of Variance

Source        DF          SS          MS         F        P
Regression     1       33672       33672     37.94    0.000
Error         14       12426         888
Total         15       46098
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by p-value ≅ 0.000.



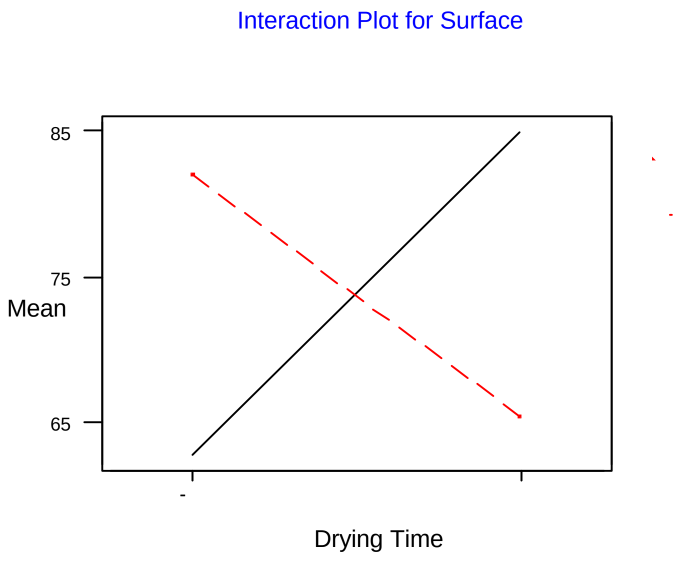


There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The assumption of constant variance does not appear to be violated. The residuals appear to have the same spread for both levels of Temperature.

7-2. a)

```
Estimated Effects and Coefficients for Surface

Term                Effect       Coef  StDev Coef        T       P
Constant                       73.750       3.955    18.65   0.000
Paint               -0.167     -0.083       3.955    -0.02   0.984
Drying               2.833      1.417       3.955     0.36   0.729
Paint*Drying       -19.500     -9.750       3.955    -2.47   0.039
```

b)



The interaction plot indicates an interaction between drying time and paint type.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that the interaction of Paint type*Drying Time is significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate ± 2(s.e.(effect))
where s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Paint Type: s.e.(effect) = 2(3.955) = 7.91
Approximate 95% confidence interval on the the effect of Paint Type:

–0.167 ± 2(7.91)
(–15.987, 15.653)

Drying Time: s.e.(effect) = 2(3.955) = 7.91
Approximate 95% confidence interval for the effect of Drying Time:

2.833 ± 2(7.91)
(–12.987, 18.653)

Paint Type*Drying Time: s.e.(effect) = 2(3.955) = 7.91
Approximate 95% confidence interval for the effect of Paint Type*Drying Time:

–19.50 ± 2(7.91)
(–35.32, –3.68)

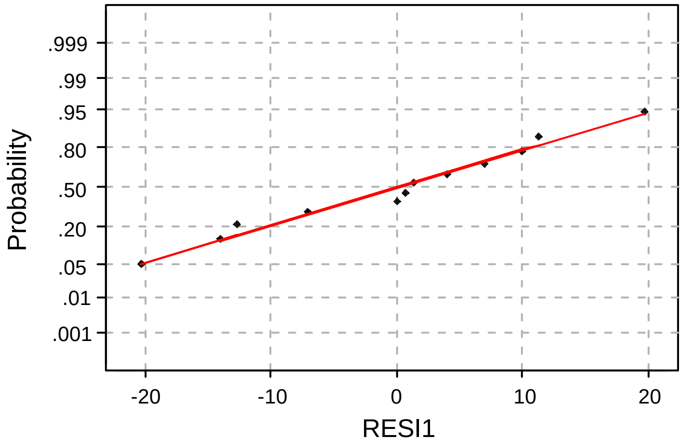
e) The regression analysis and, in this case, the final model are

```
The regression equation is
Surface Finish = 73.8 - 0.08 Paint Type + 1.42 Drying Time
                - 9.75 Paint Type*Drying Time

Predictor       Coef       StDev          T        P
Constant      73.750       3.955      18.65    0.000
Paint Ty      -0.083       3.955      -0.02    0.984
Drying T       1.417       3.955       0.36    0.729
Paint*Drying  -9.750       3.955      -2.47    0.039

S = 13.70       R-Sq = 43.7%     R-Sq(adj) = 22.6%
```

Based on the regression analysis, only the interaction appears to be significant. Since we have adopted the procedure of using hierarchical models, then this result is equivalent to that obtained in part c.

```
Analysis of Variance

Source       DF          SS          MS         F        P
Regression    3      1164.9       388.3      2.07    0.183
Error         8      1501.3       187.7
Total        11      2666.2

Source       DF      Seq SS
Paint Ty      1         0.1
Drying T      1        24.1
Paint Ty      1      1140.7
```

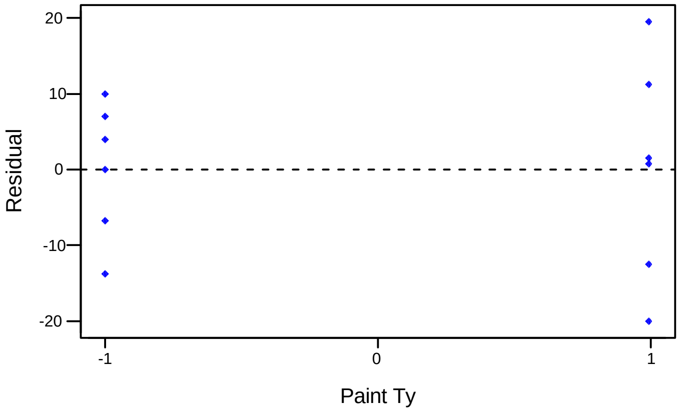
The analysis of variance may indicate the final regression model is inadequate for this set of data. The adequacy may be influenced by the inclusion of the insignificant main effects to maintain hierarchy.



Normal Probability Plot

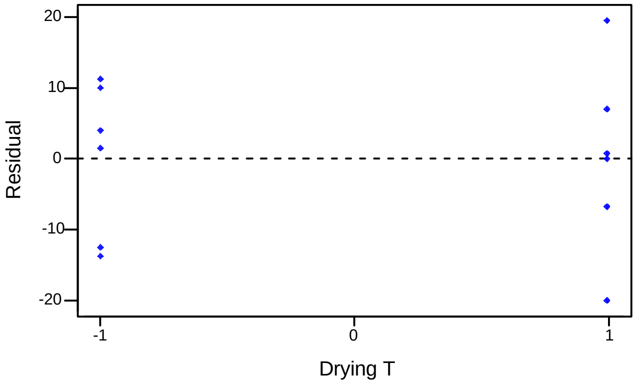
Average: -0.0000000
StDev: 11.6827
N: 12

Anderson-Darling Normality Test
A-Squared: 0.200
P-Value: 0.846



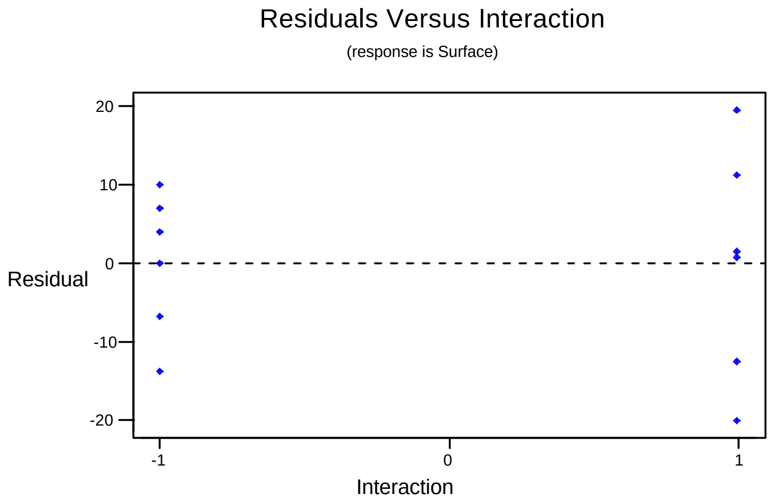
Residuals Versus Paint Ty

(response is Surface)



Residuals Versus Drying T

(response is Surface)

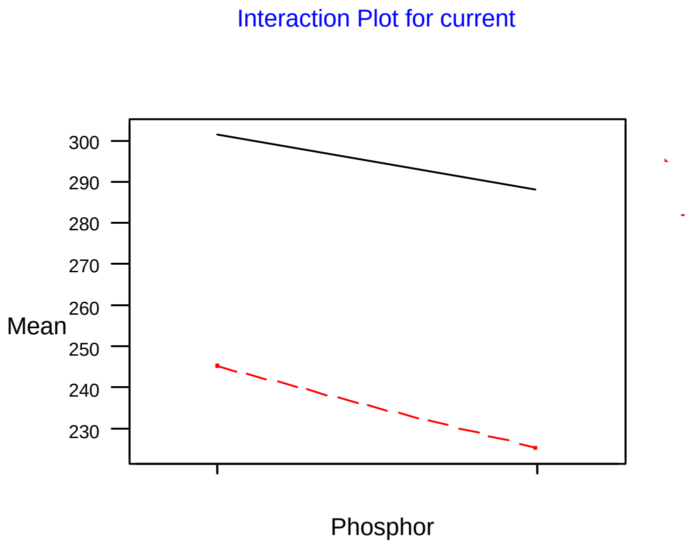


There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The assumption of constant variance does not appear to be seriously violated. The residuals for the high level of the interaction appear to be slightly more spread out than those for the low level.

7-3. a)

```
Estimated Effects and Coefficients for current

Term                 Effect      Coef  StDev Coef        T       P
Constant                       265.00       2.357   112.43   0.000
Glass                -60.00    -30.00       2.357   -12.73   0.000
Phosphor             -16.67     -8.33       2.357    -3.54   0.008
Glass*Phosphor        -3.33     -1.67       2.357    -0.71   0.500
```

b)



The interaction plot does not indicate any significant interaction between Glass and Phosphor type.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that both Glass type and Phosphor type are significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate ± 2(s.e.(effect))
where s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Glass Type: s.e.(effect) = 2(2.357) = 4.714
Approximate 95% confidence interval on the the effect of Glass Type:

–60.0 ± 2(4.714)
(–69.43, –50.57)

Phosphor Type: s.e.(effect) =2(2.357) = 4.714
Approximate 95% confidence interval for the effect of Phosphor Type:

–16.67± 2(4.714)
(–26.1, –7.242)

Glass Type*Phosphor Type: s.e.(effect) =2(2.357) = 4.714
Approximate 95% confidence interval for the effect of Glass Type*Phosphor Type:

–3.33 ± 2(4.714)
(–12.76, –6.1)

e)

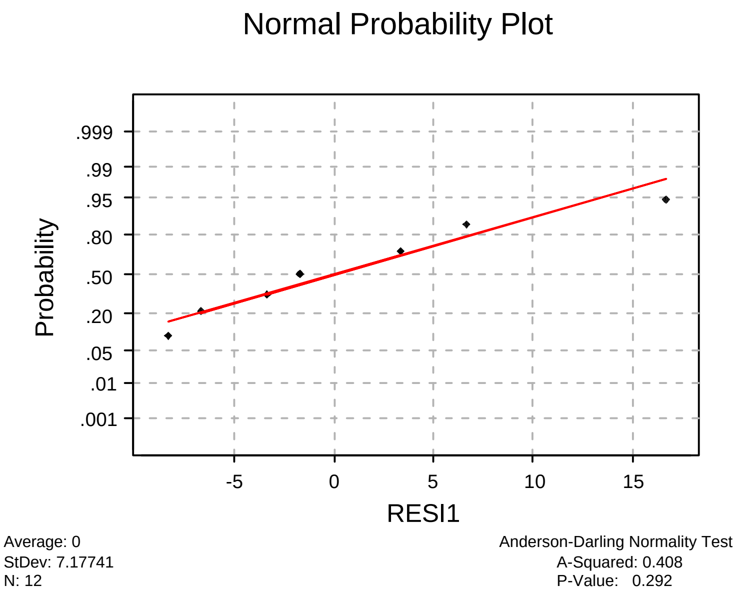
```
The regression equation is
current = 265 - 30.0 Glass - 8.33 Phosphor - 1.67 Glass*Phosphor

Predictor       Coef       StDev          T        P
Constant     265.000       2.357     112.43    0.000
Glass        -30.000       2.357     -12.73    0.000
Phosphor      -8.333       2.357      -3.54    0.008
Glass*Ph      -1.667       2.357      -0.71    0.500
```

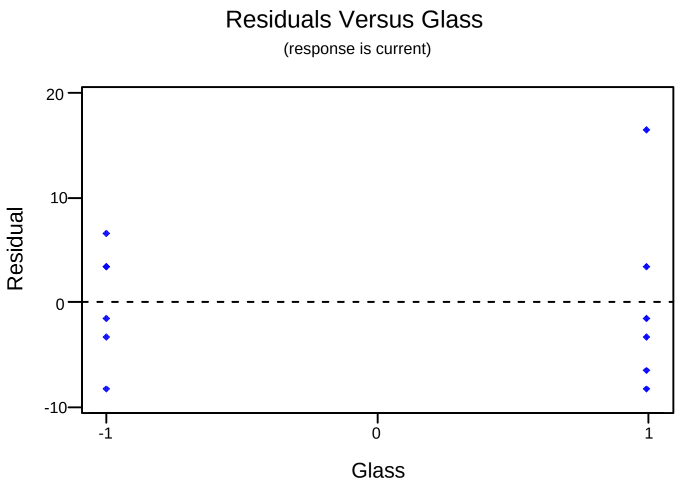
Based on the regression analysis, both Glass type and Phosphor type appear to be the significant factors. This result is equivalent to that obtained in part c.

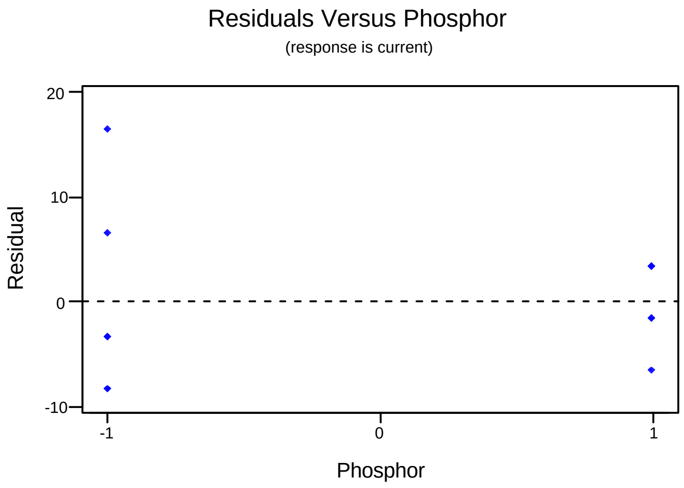
The regression analysis and final model are:

```
The regression equation is
current = 265 - 30.0 Glass - 8.33 Phosphor

Predictor       Coef       StDev          T        P
Constant     265.000       2.291     115.69    0.000
Glass        -30.000       2.291     -13.10    0.000
Phosphor      -8.333       2.291      -3.64    0.005

S = 7.935       R-Sq = 95.4%     R-Sq(adj) = 94.3%

Analysis of Variance

Source       DF          SS          MS         F        P
Regression    2     11633.3      5816.7     92.38    0.000
Error         9       566.7        63.0
Total        11     12200.0

Source       DF      Seq SS
Glass         1     10800.0
Phosphor      1       833.3
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by p-value ≅ 0.000.


Normal Probability Plot
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
-5
0
5
10
15
RESI1
Average: 0
StDev: 7.17741
N: 12
Anderson-Darling Normality Test
A-Squared: 0.408
P-Value: 0.292


Residuals Versus Glass
(response is current)
Residual
20
10
0
-10
-1
0
1
Glass

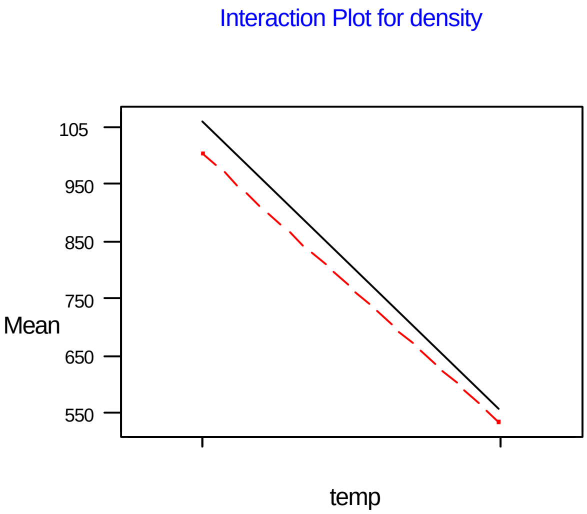

Residuals Versus Phosphor
(response is current)
Residual
20
10
0
-10
-1
0
1
Phosphor

There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. As for constant variance the residuals appear to be more spread out for the low level of Phosphor, while they are more spread out for the high level of Glass.

7-4. a)

```
Estimated Effects and Coefficients for density

Term               Effect      Coef  StDev Coef       T      P
Constant                      788.8       7.098  111.13  0.000
position            -39.5     -19.8       7.098   -2.78  0.024
temp               -490.5    -245.2       7.098  -34.55  0.000
position*temp        16.5       8.3       7.098    1.16  0.279
```

b)



The interaction plot does not indicate a significant interaction between temperature and position.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that both Position and Temperature are significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate ± 2(s.e.(effect))
where s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Position: s.e.(effect) = 2(7.098) = 14.196
Approximate 95% confidence interval on the the effect of Position:

–39.5 ± 2(14.196)
(–67.89, –11.11)

Temperature: s.e.(effect) =2(7.098) = 14.196
Approximate 95% confidence interval for the effect of Temperature:

–490.5± 2(14.196)
(–518.89, –462.11)

Position*Temperature: s.e.(effect) =2(7.098) = 14.196
Approximate 95% confidence interval for the effect of Position*Temperature:

16.5 ± 2(14.196)
(–11.89, 44.89)

e)

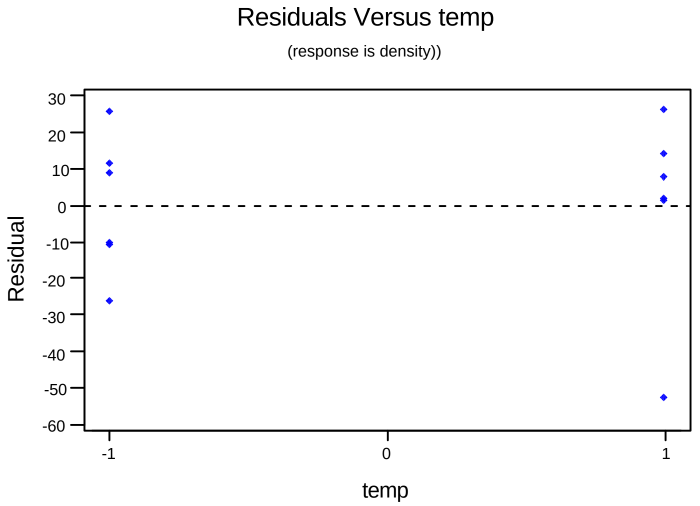
```
The regression equation is
density = 789 - 19.7 position - 245 temp + 8.25 Position*Temp

Predictor       Coef       StDev          T        P
Constant     788.750       7.098     111.13    0.000
position     -19.750       7.098      -2.78    0.024
temp        -245.250       7.098     -34.55    0.000
Position*Temp  8.250       7.098       1.16    0.279
```

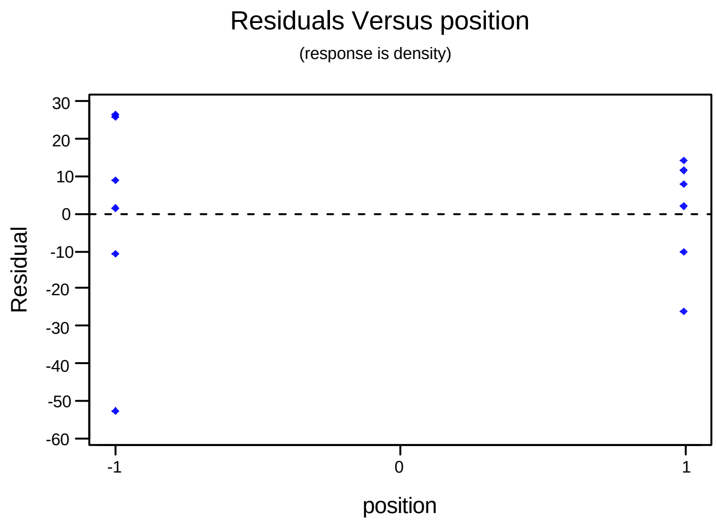
Based on the regression analysis, both Position and Temperature appear to be the significant factors. This result is equivalent to that obtained in part c.

The regression analysis and final model are

```
The regression equation is
density = 789 - 19.7 position - 245 temp

Predictor       Coef       StDev          T        P
Constant     788.750       7.235     109.02    0.000
position     -19.750       7.235      -2.73    0.023
temp        -245.250       7.235     -33.90    0.000

S = 25.06       R-Sq = 99.2%     R-Sq(adj) = 99.1%

Analysis of Variance

Source       DF          SS          MS         F        P
Regression    2      726451      363226    578.31    0.000
Error         9        5653         628
Total        11      732104

Source       DF      Seq SS
position      1        4681
temp          1      721771
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by p-value ≅ 0.000.




Residuals Versus position

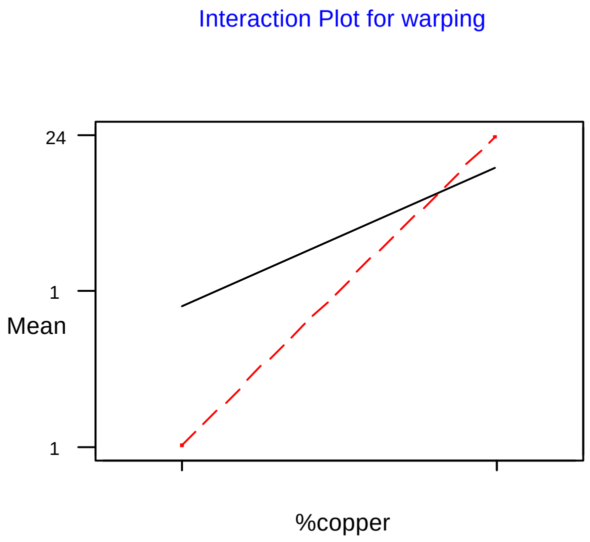
(response is density)

There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The assumption of constant variance does not appear to be violated. The residuals appear to have approximately the same spread for both levels of Temperature and both levels of Position.

7-5. a)

```
Estimated Effects and Coefficients for warping

Term              Effect     Coef  StDev Coef      T      P
Constant                  19.8750      0.7181  27.68  0.000
temp             -1.7500  -0.8750      0.7181  -1.22  0.290
%copper           7.2500   3.6250      0.7181   5.05  0.007
temp*%copper      2.7500   1.3750      0.7181   1.91  0.128
```

b)


Interaction Plot for warping

The interaction plot indicates there may be a slightly significant interaction between %Copper and Temperature.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that only %Copper is significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate ± 2(s.e.(effect))
where s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Temperature: s.e.(effect) = 2(0.7181) = 1.436
Approximate 95% confidence interval on the the effect of Temperature:

−1.75 ± 2(1.436)
(−4.622,1.122)

%Copper: s.e.(effect) =2(0.7181) = 1.436
Approximate 95% confidence interval for the effect of %Copper:

7.25± 2(1.436)
(4.378, 10.122)

Temperature*%Copper: s.e.(effect) =2(0.7181) = 1.436
Approximate 95% confidence interval for the effect of Temperature*%Copper:

2.75 ± 2(1.436)
(−0.122, 5.622)

e)

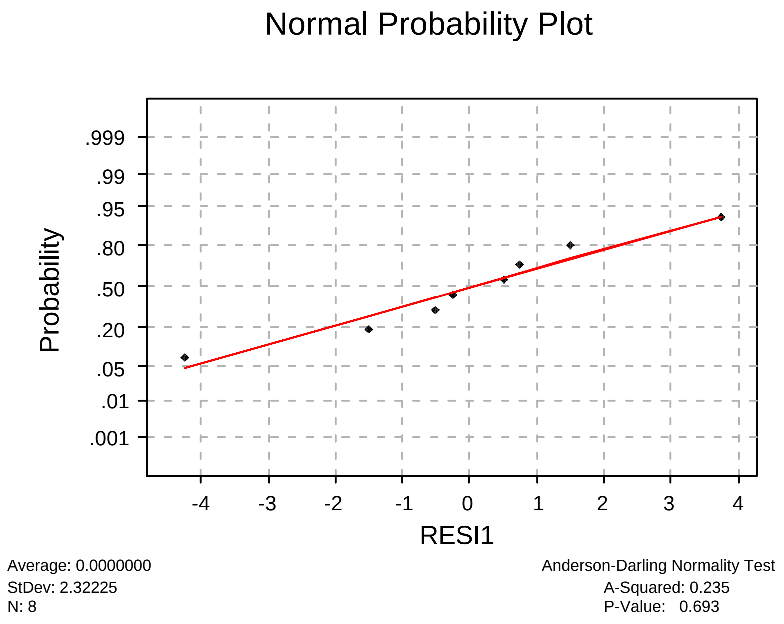
```
The regression equation is
warping = 19.9 - 0.875 temp + 3.63 %copper + 1.38 Temp*%Copper

Predictor       Coef       StDev          T        P
Constant     19.8750      0.7181      27.68    0.000
temp         -0.8750      0.7181      -1.22    0.290
%copper       3.6250      0.7181       5.05    0.007
Temp*%Co      1.3750      0.7181       1.91    0.128
```

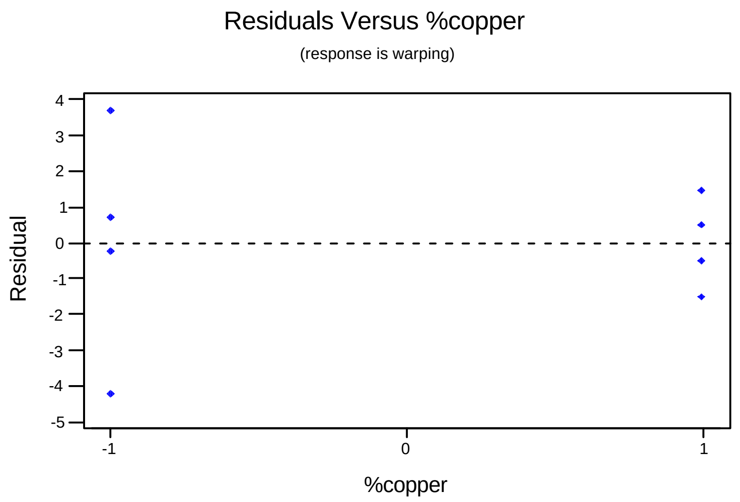
Based on the regression analysis, only %Copper appears to be the significant factor. This result is equivalent to that obtained in part c.

The regression analysis and final model are

```
The regression equation is
warping = 19.9 + 3.62 %copper

Predictor       Coef       StDev          T        P
Constant     19.8750      0.8868      22.41    0.000
%copper       3.6250      0.8868       4.09    0.006

S = 2.508       R-Sq = 73.6%     R-Sq(adj) = 69.2%

Analysis of Variance

Source       DF          SS          MS         F        P
Regression    1      105.13      105.13     16.71    0.006
Error         6       37.75        6.29
Total         7      142.88
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by p-value ≅ 0.006.



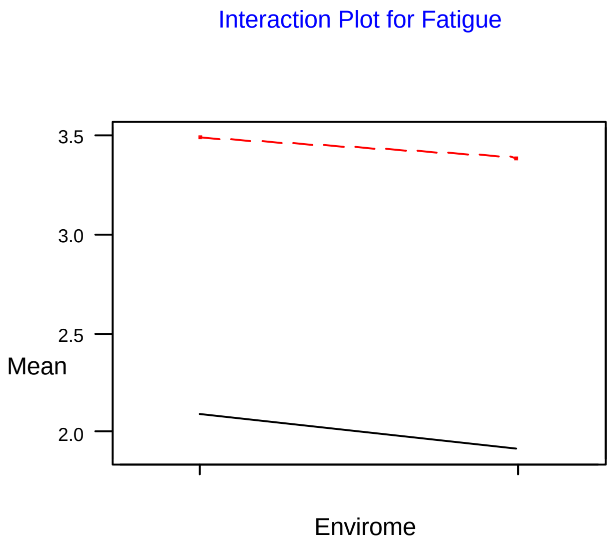


There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The residuals for the low level of %Copper are more spread out than for the high level. The constant variance assumption may be a concern.

7-6. a)

```
Estimated Effects and Coefficients for Fatigue

Term                    Effect      Coef  StDev Coef       T      P
Constant                         2.72187     0.07126   38.20  0.000
Frequenc               1.44125   0.72063     0.07126   10.11  0.000
Envirome              -0.14375  -0.07187     0.07126   -1.01  0.333
Frequenc*Envirome      0.03875   0.01937     0.07126    0.27  0.790
```

b)



The interaction plot does not indicate a significant interaction between Enviroment and Frequency.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that only Frequency is significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate ± 2(s.e.(effect))
where s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Frequency: s.e.(effect) = 2(0.07126) = 0.1425
Approximate 95% confidence interval on the the effect of Frequency:

1.44125 ± 2(0.1425)
(1.299,1.584)

Enviroment: s.e.(effect) =2(0.07126) = 0.1425
Approximate 95% confidence interval for the effect of Enviroment:

–.14375± 2(0.1425)
(–0.428, 0.142)

Frequency*Enviroment: s.e.(effect) =2(0.07126) = 0.1425
Approximate 95% confidence interval for the effect of Frequency*Enviroment:

0.03875 ± 2(0.1425)
(–0.246, 0.324)

e)

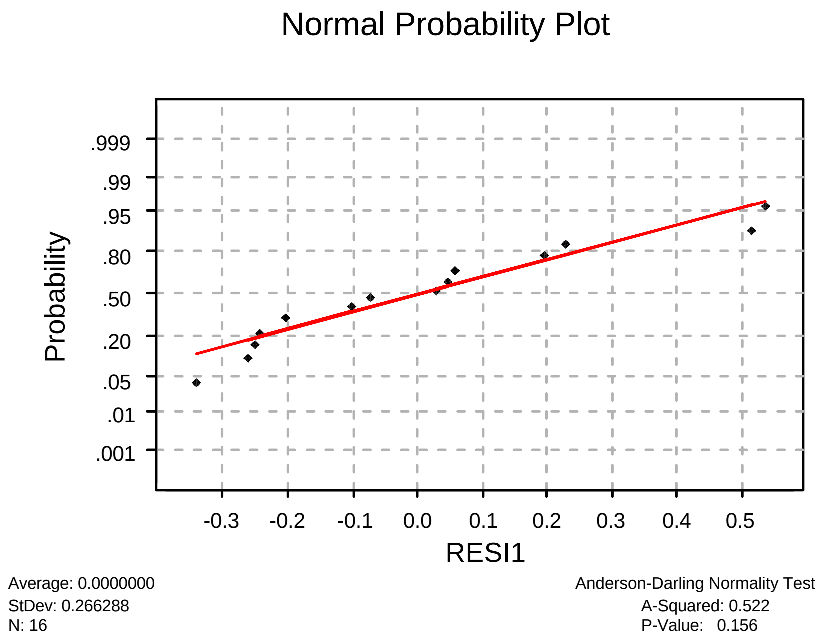
```
The regression equation is
Fatigue = 2.72 + 0.721 Frequency - 0.0719 Enviroment + 0.0194
Freq*Envir

Predictor        Coef       StDev          T        P
Constant      2.72187     0.07126      38.20    0.000
Frequenc      0.72063     0.07126      10.11    0.000
Envirome     -0.07187     0.07126      -1.01    0.333
Freq*Env      0.01938     0.07126       0.27    0.790
```

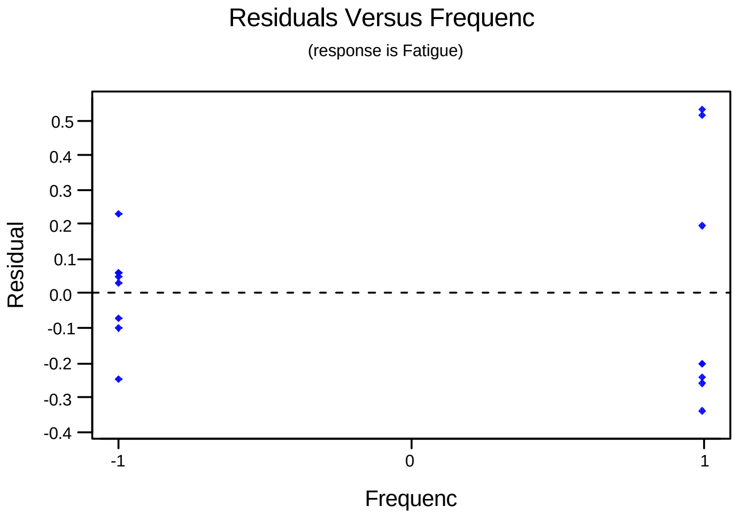
Based on the regression analysis, only Frequency appears to be the significant factor. This result is equivalent to that obtained in part c.

The regression analysis and final model are

```
The regression equation is
Fatigue = 2.72 + 0.721 Frequency

Predictor       Coef       StDev          T        P
Constant     2.72187     0.06891      39.50    0.000
Frequenc     0.72063     0.06891      10.46    0.000

S = 0.2756      R-Sq = 88.7%     R-Sq(adj) = 87.8%

Analysis of Variance

Source       DF          SS          MS         F        P
Regression    1      8.3088      8.3088    109.36    0.000
Error        14      1.0636      0.0760
Total        15      9.3724

Unusual Observations
Obs  Frequenc    Fatigue        Fit  StDev Fit   Residual    St Resid
 10      1.00     3.9600     3.4425     0.0975     0.5175       2.01R
 12      1.00     3.9800     3.4425     0.0975     0.5375       2.08R

R denotes an observation with a large standardized residual
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by p-value ≅ 0.000.



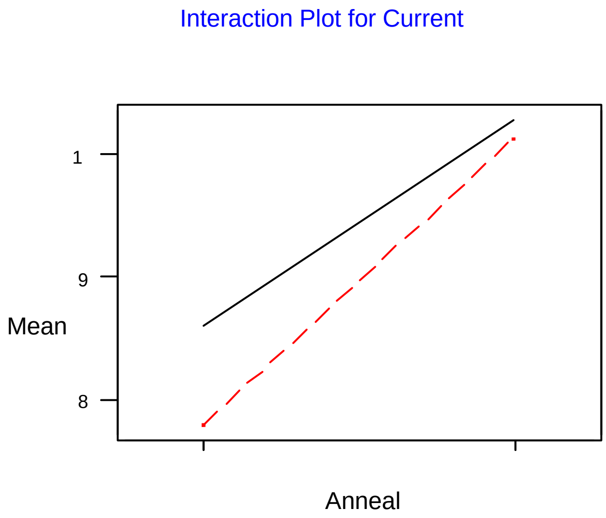


There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The residuals for the high level of Frequency are more spread out than for the low level. The constant variance assumption may be a concern.

7-7. a)

```
Estimated Effects and Coefficients for Current

Term                  Effect      Coef   StDev Coef        T       P
Constant                        9.2050      0.07622   120.77   0.000
Polysili             -0.4850   -0.2425      0.07622    -3.18   0.033
Anneal                2.0300    1.0150      0.07622    13.32   0.000
Polysili*Anneal       0.3350    0.1675      0.07622     2.20   0.093
```

b)



The interaction plot does not indicate a significant interaction between Polysilicon Doping and Anneal.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that both Polysilicon doping and Anneal are significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate ± 2(s.e.(effect)) where s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Polysilicon Doping: s.e.(effect) = 2(0.07622) = 0.1524
Approximate 95% confidence interval on the the effect of Polysilicon Doping:

−0.485± 2(0.1524)
(−0.7898, −0.1802)

Anneal: s.e.(effect) =2(0.07622) = 0.1524
Approximate 95% confidence interval for the effect of Anneal:

2.03± 2(0.1524)
(1.725, 2.34)

Polysilicon doping*Anneal: s.e.(effect) =2(0.07622) = 0.1524
Approximate 95% confidence interval for the effect of Polysilicon doping*Anneal:

0.335 ± 2(0.1524)
(0.0302, 0.6398)

e)

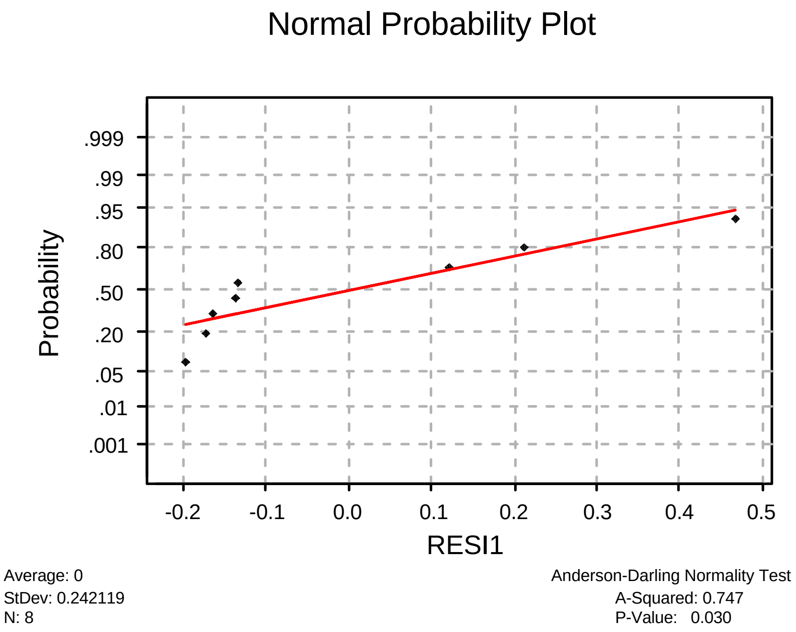
```
The regression equation is
Current = 9.21 - 0.242 Polysilicon + 1.02 Anneal + 0.167 Poly*Anneal

Predictor       Coef       StDev           T        P
Constant     9.20500     0.07622     120.77    0.000
Polysili    -0.24250     0.07622      -3.18    0.033
Anneal       1.01500     0.07622      13.32    0.000
Poly*Ann     0.16750     0.07622       2.20    0.093
```

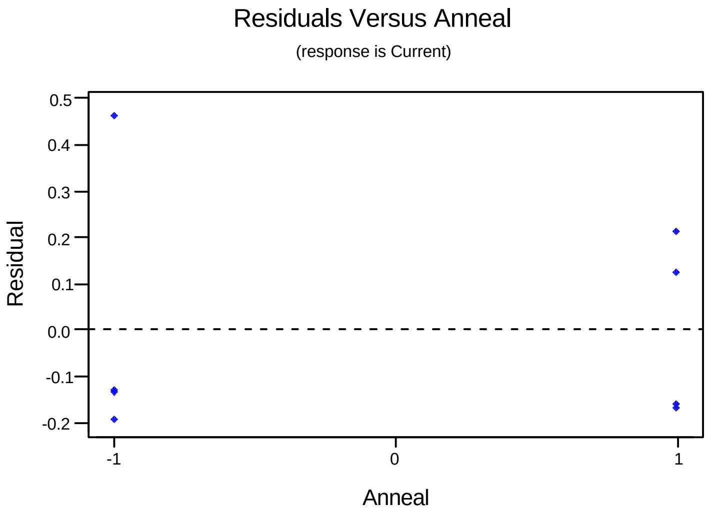
Based on the regression analysis, both Polysilicon and Anneal appear to be the significant factors. This result is equivalent to that obtained in part c.

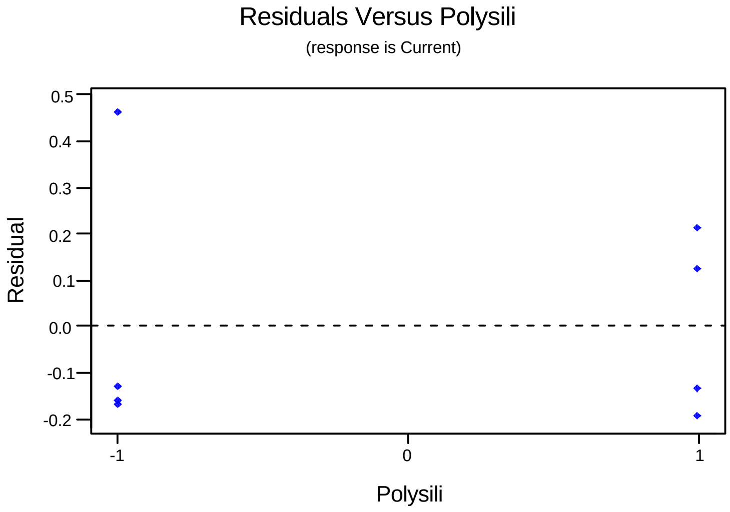
The regression analysis and final model are

```
The regression equation is
Current = 9.21 - 0.242 Polysilicon + 1.02 Anneal

Predictor       Coef       StDev           T        P
Constant      9.2050      0.1013      90.88    0.000
Polysili     -0.2425      0.1013      -2.39    0.062
Anneal        1.0150      0.1013      10.02    0.000

S = 0.2865      R-Sq = 95.5%     R-Sq(adj) = 93.7%

Analysis of Variance

Source       DF          SS          MS         F        P
Regression    2      8.7123      4.3561     53.08    0.000
Error         5      0.4103      0.0821
Total         7      9.1226

Source       DF      Seq SS
Polysili      1      0.4704
Anneal        1      8.2418

Unusual Observations
Obs  Polysili    Current        Fit  StDev Fit   Residual    St Resid
  5     -1.00      8.900      8.432      0.175      0.468       2.06R

R denotes an observation with a large standardized residual
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by p-value ≅ 0.000.


Normal Probability Plot
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
0.5
RESI1
Average: 0
StDev: 0.242119
N: 8
Anderson-Darling Normality Test
A-Squared: 0.747
P-Value: 0.030


Residuals Versus Anneal
(response is Current)
Residual
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-1
0
1
Anneal

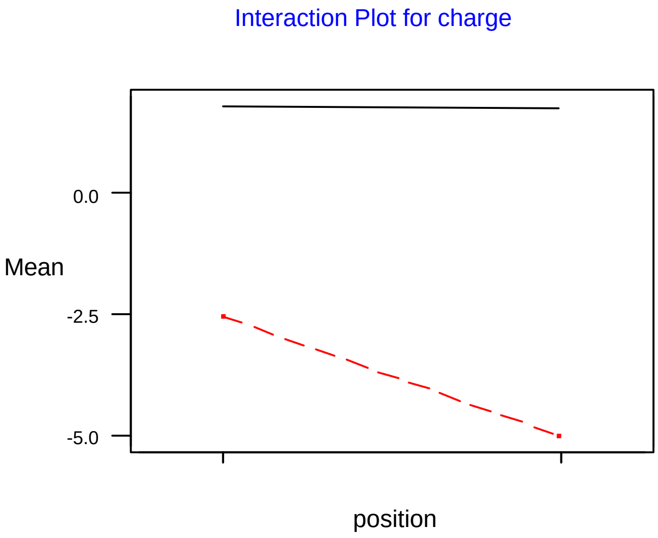


There does not appear to be any serious departure from normality shown in the normal probability plot of the residuals. The residuals for the high level of Anneal and for Polysilicon are more spread out than for the low levels of each. The constant variance assumption may be a concern.

7-8. a)

```
Estimated Effects and Coefficients for charge

Term                 Effect      Coef  StDev Coef       T       P
Constant                       -1.000      0.4462   -2.24   0.055
method               -5.593    -2.797      0.4462   -6.27   0.000
position             -1.280    -0.640      0.4462   -1.43   0.189
method*position      -1.220    -0.610      0.4462   -1.37   0.209
```

b)



The interaction plot does not indicate a significant interaction between Method and Position.

c) The t-ratios are given in the output shown in part a. The t-ratios indicate that only Method is significant at the $\alpha = 0.05$ level.

d) The 95% confidence intervals are given by effect estimate ± 2(s.e.(effect))
where s.e.(effect) = 2[s.e.(coefficient)]. The s.e.(coefficient) is given in the Minitab output of part a.

Method: s.e.(effect) = 2(0.4462) = 0.8924
Approximate 95% confidence interval on the the effect of Method:

–5.593± 2(0.8924)
(–7.38, –0.3.81)

Position: s.e.(effect) = 2(0.4462) = 0.8924
Approximate 95% confidence interval for the effect of Position:

–1.28± 2(0.8924)
(–3.065, 0.505)

Method*Position: s.e.(effect) = 2(0.4462) = 0.8924
Approximate 95% confidence interval for the effect of Method*Position:

–1.22 ± 2(0.8924)
(–3.005, 0.565)

e)

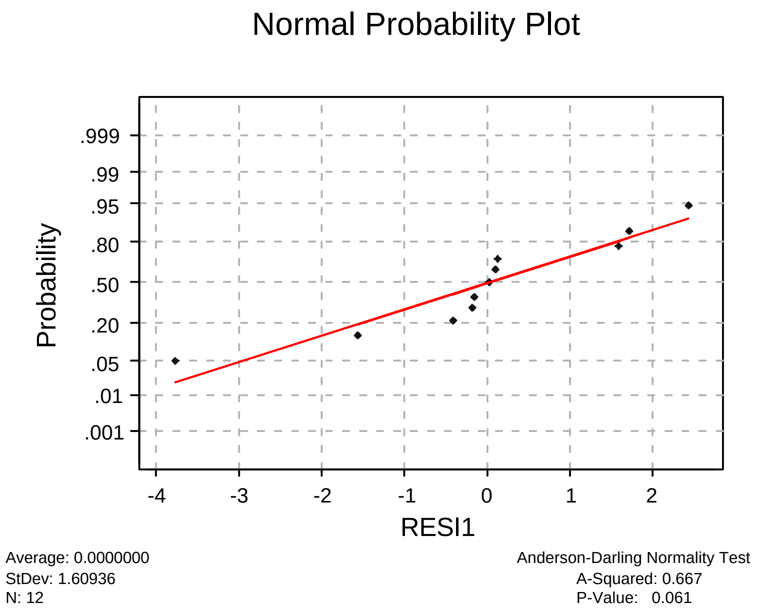
```
The regression equation is
charge = - 1.00 - 2.80 method - 0.640 position - 0.610 meth*pos

Predictor       Coef       StDev          T        P
Constant     -1.0000      0.4462      -2.24    0.055
method       -2.7967      0.4462      -6.27    0.000
position     -0.6400      0.4462      -1.43    0.189
meth*pos     -0.6100      0.4462      -1.37    0.209
```

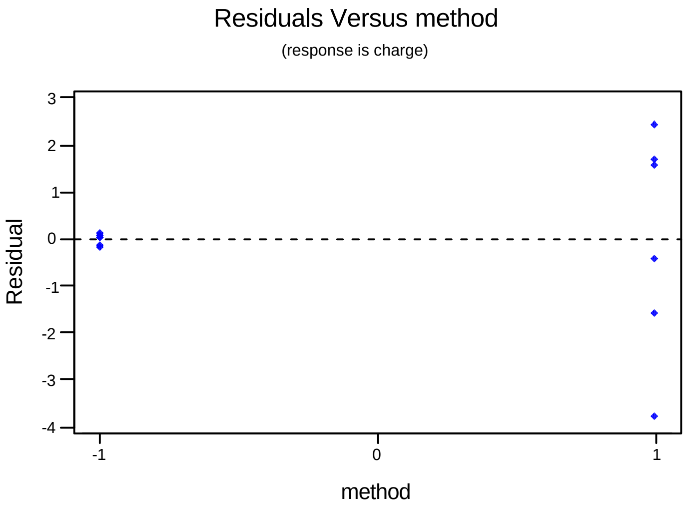
Based on the regression analysis, only Method appears to be the significant factor. This result is equivalent to that obtained in part c.

The regression analysis and final model are

```
The regression equation is
charge = - 1.00 - 2.80 method

Predictor       Coef       StDev          T        P
Constant     -1.0000      0.4873      -2.05    0.067
method       -2.7967      0.4873      -5.74    0.000

S = 1.688       R-Sq = 76.7%     R-Sq(adj) = 74.4%

Analysis of Variance

Source       DF          SS          MS         F        P
Regression    1      93.856      93.856     32.94    0.000
Error        10      28.490       2.849
Total        11     122.347

Unusual Observations
Obs    method     charge        Fit  StDev Fit   Residual    St Resid
  4      1.00     -7.580     -3.797      0.689     -3.783      -2.46R

R denotes an observation with a large standardized residual
```

The analysis of variance indicates the final regression model is adequate for this set of data. This is evident by P-value ≅ 0.000.





There appears to be a departure from normality shown in the normal probability plot of the residuals. The residuals for the high level of method are more spread out than for the low level. The constant variance assumption may be a concern.