What is Probability?

PROBABILITY

- If I make everything predictable, human beings will have no motive to do anything since the future is totally determined.
- If I make everything unpredictable, human beings will have no motive to do anything as there is no rational basis for any decision.
- I must therefore create a mixture of two.

[from E. F. Schumacher] LORD

PROBABILITY

- Deals with uncertainty.
- Plays an important role in decision making in day-to-day activities.
- There is no statistics without probability.
- Key people: Fermat, Pascal, Bernoulli, Laplace, and Kolmogorov.

HISTORY OF PROBABILITY

- **Believed to have been started by Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665) primarily in games of chance.**
- **Jacob Bernoulli, Nicholas Bernoulli, Abraham de Moivre, Pierre de Laplace, developed the theory in a much general set up.**

HISTORY OF PROBABILITY (cont'd)

- During the 19th century the French school and the Russian School were very influential in the development as we see now.
- Kolmogorov in 1933 introduced the axiomatic approach to probability theory and random processes.
- Classical books: P. Levy, H. Cramer, B.V. Gnedenko, M. Loeve, and W. Feller.

PROBABILITY

- We can compare Probability to Physics and Astronomy.
- Probabilities Masses.
- Motion of the planets can be discussed without the knowledge of individual masses and so does probabilities of events without observing them.

PROBABILITY (cont'd)

- Experiments go beyond coin tossing, picking cards, throwing dice, etc.
- Probability helps us to understand better the events surrounding us.
- Probability helps manufacturers to build quality products through understanding the needs of customers, competition, etc.

PROBABILITY
Uncertainty

- Look around and see most things in life have uncertainty. We accept some uncertainty with no real concern.
- Weather, time to reach school (work), prices of goods, regular fluctuation in stocks, etc
- Breakdown of cars, outage of electricity or gas, crash of stock markets, etc.

PROBABILITY (Decision making)

- How do the insurance companies determine the premiums?
- How do the manufacturing companies determine the warranty period?
- How do the manufacturers decide on the number of units to make?
- How do the supermarkets decide on the number of counters to open?

PROBABILITY
(Decision making - cont'd)

- How do the package delivery companies offer the guarantee and charge?
- How do the package delivery companies schedule their drivers, fleet, etc?
- How do the airlines schedule their crew, fleet, etc?
- How the jury is selected?
- How do the casinos determine the pay out for the odds in a bet?

PROBABILITY
(Decision making - cont'd)

- **Why is that, if you go to a bank or post office, you see there is only one queue in front of many tellers?**
- **Why is that, in super markets, you see several (parallel) queues?**
- **Have you ever wondered, when you call your friend over the phone, how inspite of not having a "direct" connection, you get connected without delay?**

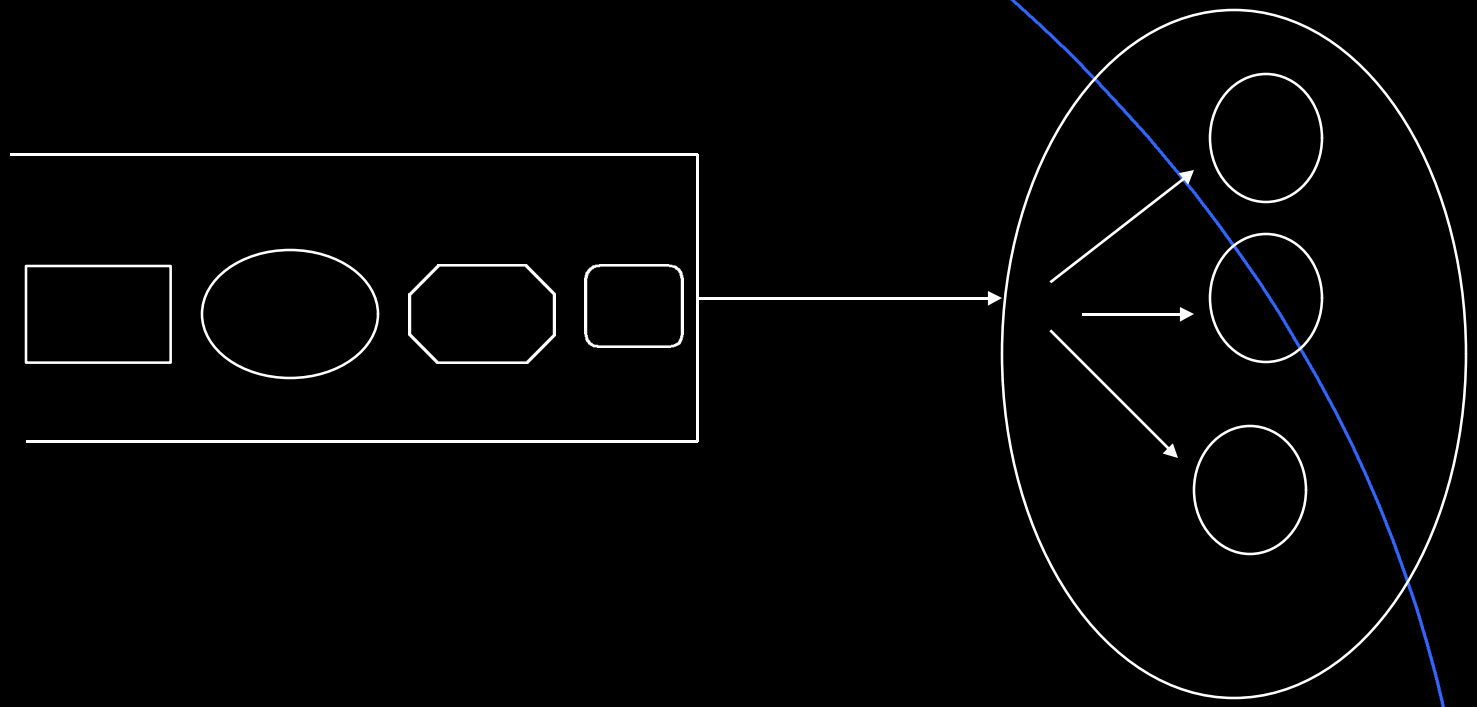
PROBABILITY
(Decision making - cont'd)

- **DNA matching (especially in crime related activities) is important in a judicial process. Have you wondered how probability plays a role here?**
- **Do you (or your parents) know how FDA monitors the consumers' interests?**
- **Classification of items or objects is a fundamental concept not only in day-to-day life, but also in other areas of science.**

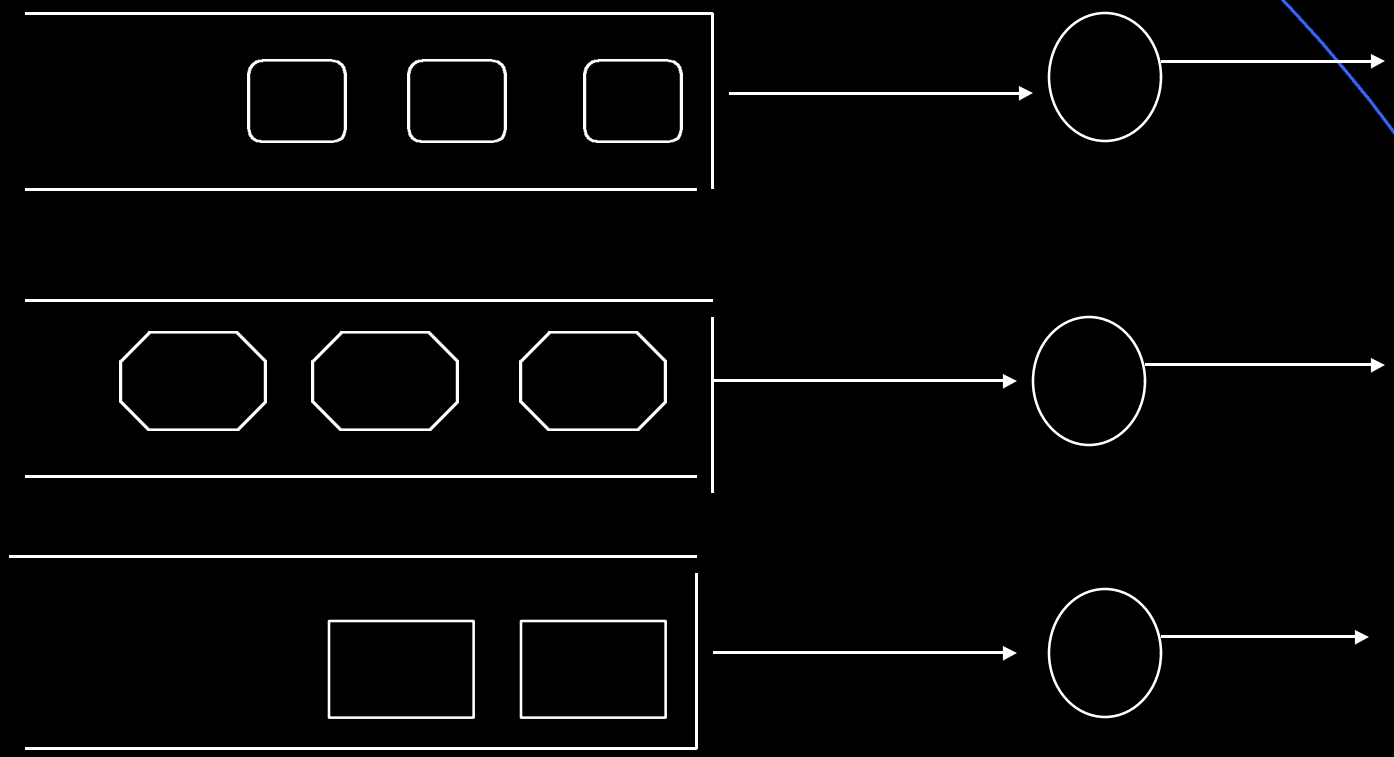
PROBABILITY
(Decision making - cont'd)

- In manufacturing systems, there are several machines that perform a set of operations to produce a final "product"
- Companies would like to have the machines that are very flexible and versatile so as to make more products and hence more profit.

PROBABILITY-Decision (FUNCTIONAL SYSTEM)



PROBABILITY-Decision (CELLULAR SYSTEM)



PROBABILITY & STATISTICS

- **Medicine and Health: Study of patient response to a new painkiller.**
- **Business and Economics: Prediction of manpower for a particular employment sector.**
- **Education: Study of correlation between SAT and academic performance**

PROBABILITY & STATISTICS

- **Biology: Selection of those plants that will be the parents of next generation improving certain characteristics.**

- **Sociology: Jury selection(how to avoid unfavorable jurors); effect of working two-parent family on their children; employment discrimination in workforce.**

PROBABILITY & STATISTICS

- **Environment: Effect on air pollutants on living things (human, plants, animals).**
- **Sports: Effect of athletes' performance on their salary structure, and entrance fees for the games.**
- **Engineering: MPG of a new model car on the size, features, etc of the car.**

PROBABILITY & STATISTICS

- **Smoke-free environment due to statistical study indicating a strong correlation between cigarette smoking and lung-related diseases, high blood pressure, etc.**
- **Passenger restrain systems (seat belts, airbags, etc) in cars as laws to minimize crash injuries.**

IMPACT OF PROBABILITY & STATISTICS

- **Effective measures to minimize the greenhouse effect.**
- **Use of statistical process control to identify the sources of problems and correct them.**
- **Making essay test scores (ETS) fairer.**
- **Making Jury selection fairer**
- **Understand discrimination in workforce**

EXAMPLE 1

- A leading package delivery company was interested in knowing how best to allocate their resources (trucks, drivers, etc) so as to improve their productivity.
- Probability modeling was used to tell them how many drivers, trucks need to be allocated to various division for M-W.

EXAMPLE 2

- Painting process in a car manufacturing plant is really fascinating! You all should go and visit when you get a chance.
- Several factors such as paint viscosity, bell location, spray, booth temperature, booth humidity, etc, are involved.
- Important to know which factors are important; what settings are needed to have a specified FBT and Uniformity.

What is PROBABILITY?

- **Experiment in which the outcome cannot be precisely determined.**
  - **Tossing a coin, throwing a die, picking a student, choosing 4 items from a lot**
- **Probability is a function (WHY?) taking values between 0 and 1.**
- **There are three definitions of probability and each one has its own merits and demerits.**

AXIOMATIC APPROACH

- **Satisfies a number of axioms.**
- **Useful in developing the theory of probability.**
- **Doesn’t tell us how to compute the probability of an event.**

FREQUENCY APPROACH

- **Suppose that an experiment is conducted n times. Let n(A) denote the number of times the event A occurs**
- **Intuitively it suggests that P(A) can be approximated with n(A)/n**
- **n(A)/n will approach P(A) as n approaches infinity.**
- **Useless since we have to perform the experiment.**

CLASSICAL APPROACH

- **Suppose an experiment has a finite number (N) of "equally likely" outcomes.**
  - **Tossing a fair coin; throwing a fair die; picking a student at random; choosing 4 items randomly from a lot.**
- **P(A) = number of outcomes in A / N**
- **What if the outcomes are not "equally likely"? Use weights!!!**

CLASSICAL (cont'd)

- **Probability of an "event" is calculated using counting techniques.**
- **Simulating probabilities of events will further strengthen the understanding of this concept. This is made even simpler with the advent of computers.**

APPLIED STATISTICS

- **Probability is best understood by looking through applied statistics.**
- **Range from very simple ones such as graphical display, summary statistics, and time-series plots, to sophisticated ones such as design of experiments, regression analysis, principal component analysis, and process control.**

WHY WE NEED STATISTICS ?

- **Variability is present in almost everything we do.**
- **Statistics helps to identify the source of variability**
- **Statistics helps to control the variability.**
- **Statistics helps to make scientific conclusions.**

SUCCESS OF STATISTICS

- **Successful application of statistical methods depends on the close interplay between theory and practice.**

EXAMPLE 3

- Nashua corporation (in NH) manufactures carbonless carbon paper.
- 1100 lft/min; used 3.6 lbs per 3000 sq ft.
- The operator was adjusting constantly and 3.6 lbs was high. Idea to buy a costly coating head.
- Statistics was used to determine that adjustments were made based on delayed

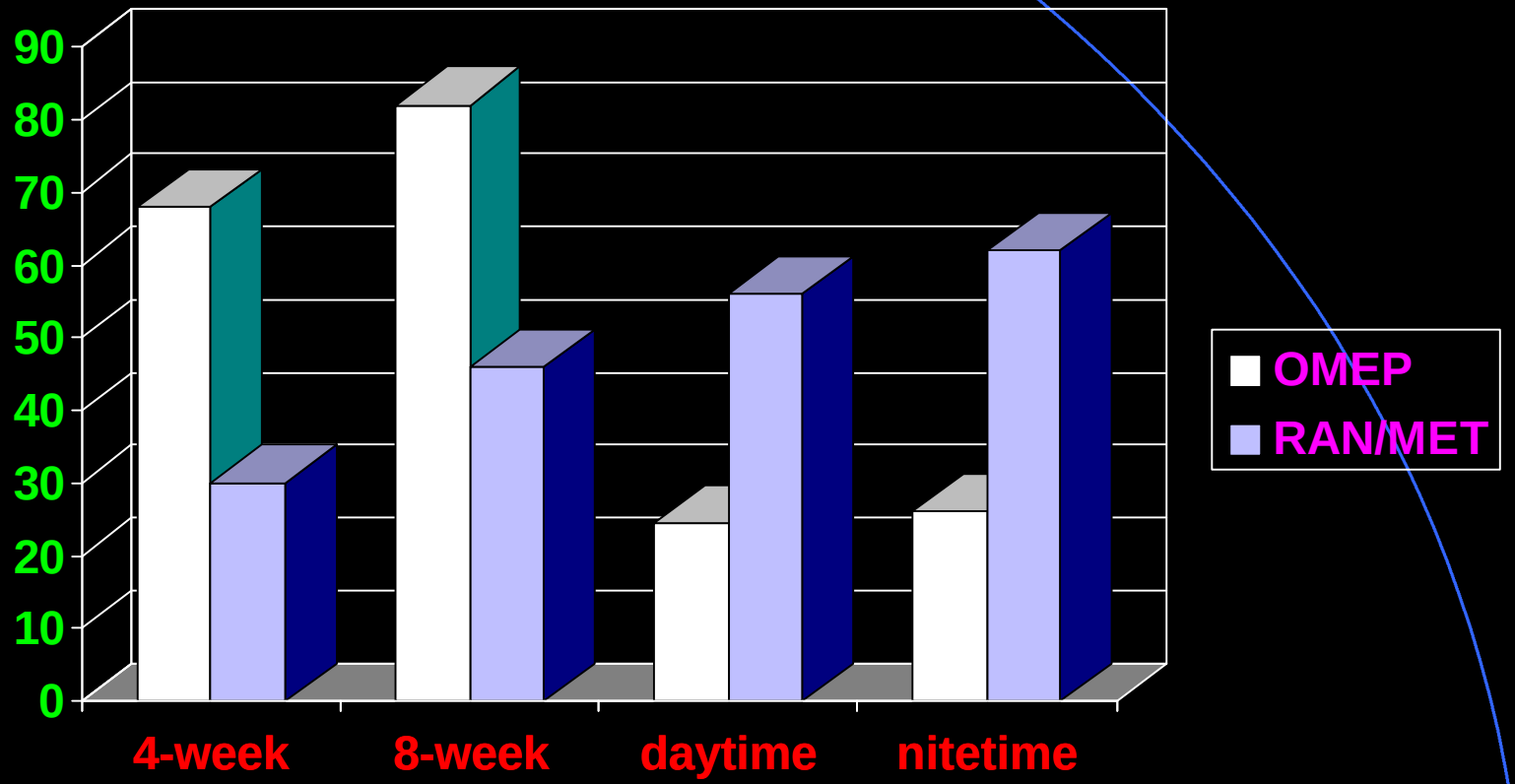
EXAMPLE 3 (cont'd)

- data and so it didn't pertain to current conditions. New operating instructions led to fewer adjustments and reduced the average to 2.6lbs of dry coating/3000 sq.ft
- Resulted in a savings of $800,000 /year in chemicals

EXAMPLE 4

- This deals with efficacy and tolerability of OMEP (omeprazole 20) vs RAN/MET (ranitidine/metoclopramide) in SEE (severe erosive esophagitis-Stomach acid in esophagus)
- Number and percentage of patients healed at 4 and 8 weeks.
- Median time to relief (days) of SEE

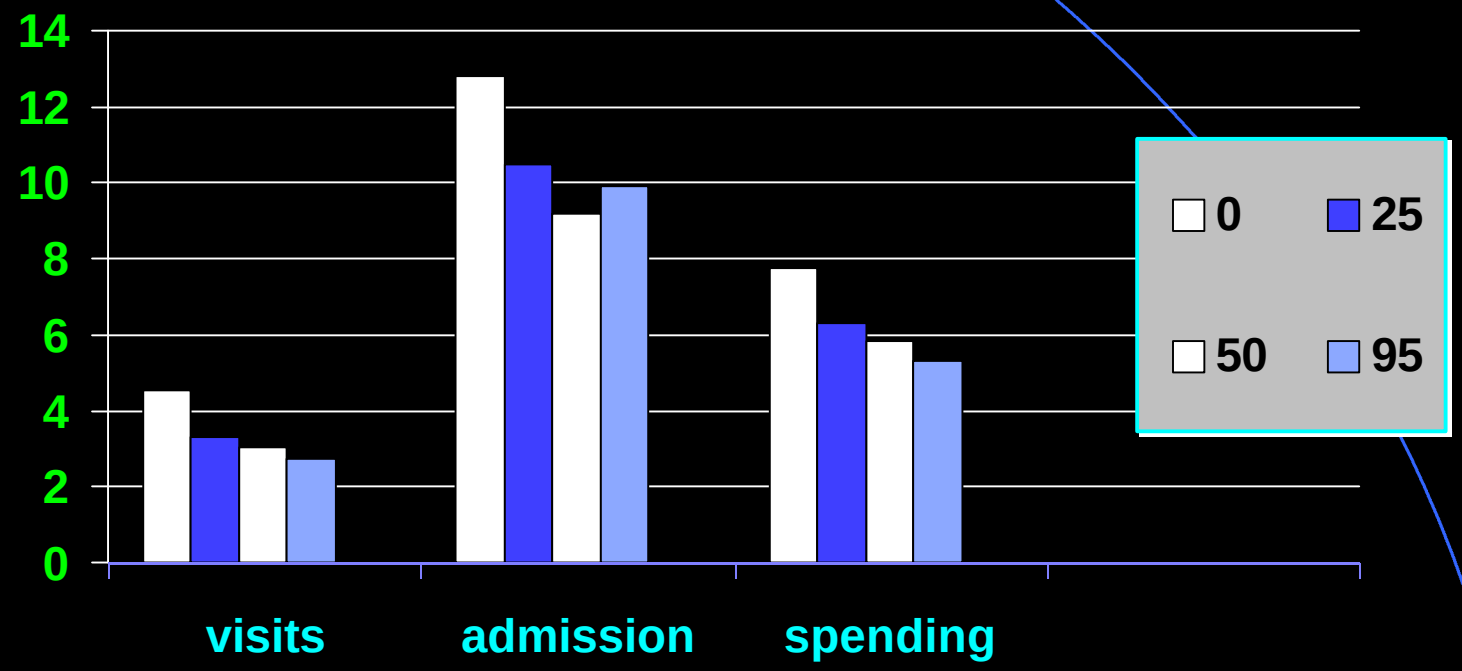
Healing rate and MTR



EXAMPLE 5

- **Health insurance: Should there be a free medical care or how much deductible and co-payment one should pay?**
- **In 1974, the federal government set up a large experiment and used statistics to see how the premium and the usage of the facilities are related.**

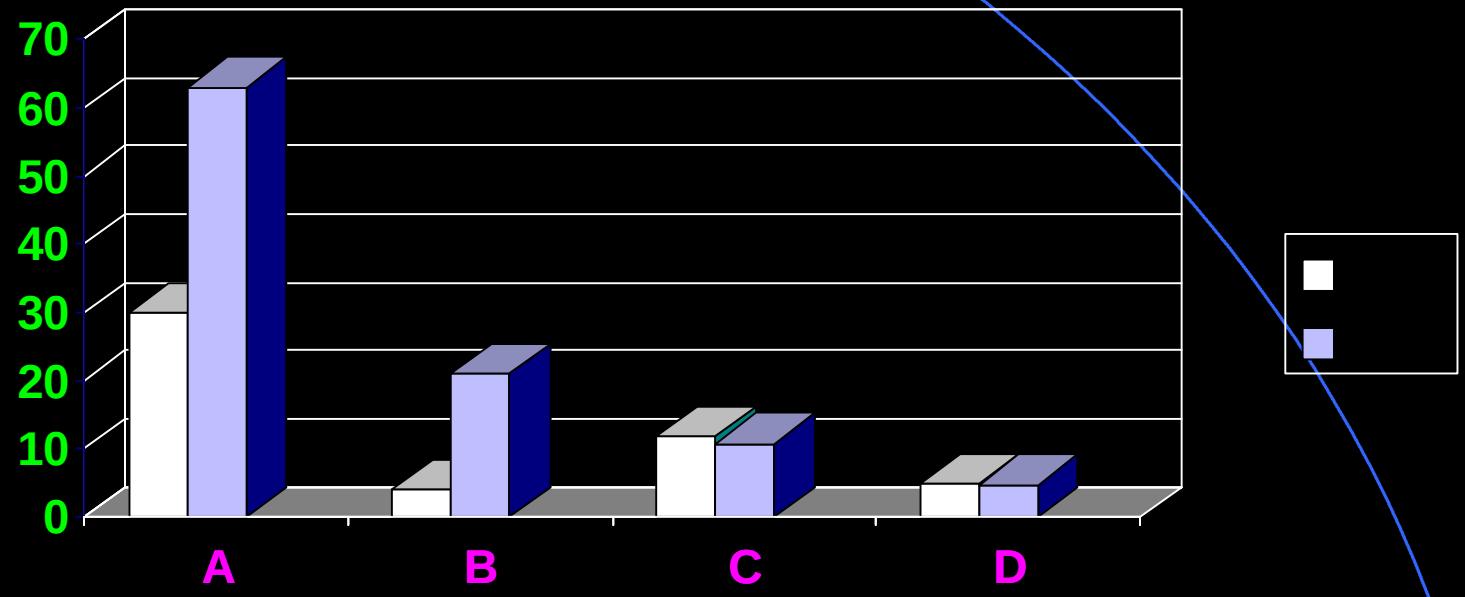
Care use vs Insurance



Analysis and Influence

- Comparing persons paying 95% of the bills and persons paying nothing, we see
- 40% fewer doctor visits
- 23% reduction in hospital usage
- 31% reduction in total spending
- Between 1982 & 1984, there was an increase in the amount of cost-sharing in private health insurance in USA

Change between 1982 & 1984



A = % of private insurance with a ded. for hospital visits
B = % of private insurance with $200 or more in ded.
C = Hospital discharges per 100; D = Doctor visits