What is Probability?
If I make everything predictable, human beings will have no motive to do anything since the future is totally determined.

If I make everything unpredictable, human beings will have no motive to do anything as there is no rational basis for any decision.

I must therefore create a mixture of two.

[from E. F. Schumacher]
PROBABILITY

- Deals with uncertainty.
- Plays an important role in decision making in day-to-day activities.
- There is no statistics without probability.
- Key people: Fermat, Pascal, Bernoulli, Laplace, and Kolmogorov.
HISTORY OF PROBABILITY

- Believed to have been started by Blaise Pascal (1623-1662) and Pierre de Fermat (1601-1665) primarily in games of chance.

- Jacob Bernoulli, Nicholas Bernoulli, Abraham de Moivre, Pierre de Laplace, developed the theory in a much general set up.
During the 19th century the French school and the Russian School were very influential in the development as we see now.

Kolmogorov in 1933 introduced the axiomatic approach to probability theory and random processes.

We can compare Probability to Physics and Astronomy.

Probabilities  Masses.

Motion of the planets can be discussed without the knowledge of individual masses and so does probabilities of events without observing them.
Experiments go beyond coin tossing, picking cards, throwing dice, etc.

Probability helps us to understand better the events surrounding us.

Probability helps manufacturers to build quality products through understanding the needs of customers, competition, etc.
PROBABILITY
Uncertainty

- Look around and see most things in life have uncertainty. We accept some uncertainty with no real concern.
- Weather, time to reach school (work), prices of goods, regular fluctuation in stocks, etc.
- Breakdown of cars, outage of electricity or gas, crash of stock markets, etc.
How do the insurance companies determine the premiums?

How do the manufacturing companies determine the warranty period?

How do the manufacturers decide on the number of units to make?

How do the supermarkets decide on the number of counters to open?
PROBABILITY
(Decision making - cont’d)

- How do the package delivery companies offer the guarantee and charge?
- How do the package delivery companies schedule their drivers, fleet, etc?
- How do the airlines schedule their crew, fleet, etc?
- How the jury is selected?
- How do the casinos determine the pay out for the odds in a bet?
Why is that, if you go to a bank or post office, you see there is only one queue in front of many tellers?

Why is that, in super markets, you see several (parallel) queues?

Have you ever wondered, when you call your friend over the phone, how inspite of not having a “direct” connection, you get connected without delay?
DNA matching (especially in crime related activities) is important in a judicial process. Have you wondered how probability plays a role here?

Do you (or your parents) know how FDA monitors the consumers’ interests?

Classification of items or objects is a fundamental concept not only in day-to-day life, but also in other areas of science.
In manufacturing systems, there are several machines that perform a set of operations to produce a final “product”

Companies would like to have the machines that are very flexible and versatile so as to make more products and hence more profit.
PROBABILITY-Decision
(FUNCTIONAL SYSTEM)
PROBABILITY-Decision
(CELLULAR SYSTEM)
PROBABILITY & STATISTICS

- Medicine and Health: Study of patient response to a new painkiller.
- Business and Economics: Prediction of manpower for a particular employment sector.
- Education: Study of correlation between SAT and academic performance
PROBABILITY & STATISTICS

- **Biology**: Selection of those plants that will be the parents of next generation improving certain characteristics.

- **Sociology**: Jury selection (how to avoid unfavorable jurors); effect of working two-parent family on their children; employment discrimination in workforce.
PROBABILITY & STATISTICS

- **Environment**: Effect on air pollutants on living things (human, plants, animals).
- **Sports**: Effect of athletes’ performance on their salary structure, and entrance fees for the games.
- **Engineering**: MPG of a new model car on the size, features, etc. of the car.
PROBABILITY & STATISTICS

- Smoke-free environment due to statistical study indicating a strong correlation between cigarette smoking and lung-related diseases, high blood pressure, etc.

- Passenger restrain systems (seat belts, airbags, etc) in cars as laws to minimize crash injuries.
IMPACT OF PROBABILITY & STATISTICS

- Effective measures to minimize the greenhouse effect.
- Use of statistical process control to identify the sources of problems and correct them.
- Making essay test scores (ETS) fairer.
- Making Jury selection fairer
- Understand discrimination in workforce
EXAMPLE 1

- A leading package delivery company was interested in knowing how best to allocate their resources (trucks, drivers, etc) so as to improve their productivity.
- Probability modeling was used to tell them how many drivers, trucks need to be allocated to various division for M-W.
EXAMPLE 2

- Painting process in a car manufacturing plant is really fascinating! You all should go and visit when you get a chance.
- Several factors such as paint viscosity, bell location, spray, booth temperature, booth humidity, etc, are involved.
- Important to know which factors are important; what settings are needed to have a specified FBT and Uniformity.
What is PROBABILITY?

- Experiment in which the outcome cannot be precisely determined.
  - Tossing a coin, throwing a die, picking a student, choosing 4 items from a lot

- Probability is a function (WHY?) taking values between 0 and 1.

- There are three definitions of probability and each one has its own merits and demerits.
AXIOMATIC APPROACH

- Satisfies a number of axioms.
- Useful in developing the theory of probability.
- Doesn’t tell us how to compute the probability of an event.
FREQUENCY APPROACH

Suppose that an experiment is conducted n times. Let n(A) denote the number of times the event A occurs.

Intuitively it suggests that P(A) can be approximated with n(A)/n.

n(A)/n will approach P(A) as n approaches infinity.

Useless since we have to perform the experiment.
Suppose an experiment has a finite number \((N)\) of “equally likely” outcomes.

- Tossing a fair coin; throwing a fair die; picking a student at random; choosing 4 items randomly from a lot.

\[
P(A) = \frac{\text{number of outcomes in } A}{N}
\]

What if the outcomes are not “equally likely”? Use weights!!!
Probability of an “event” is calculated using counting techniques.

Simulating probabilities of events will further strengthen the understanding of this concept. This is made even simpler with the advent of computers.
Probability is best understood by looking through applied statistics.

Range from very simple ones such as graphical display, summary statistics, and time-series plots, to sophisticated ones such as design of experiments, regression analysis, principal component analysis, and process control.
WHY WE NEED STATISTICS?

- Variability is present in almost everything we do.
- Statistics helps to identify the source of variability.
- Statistics helps to control the variability.
- Statistics helps to make scientific conclusions.
SUCCESS OF STATISTICS

- Successful application of statistical methods depends on the close interplay between theory and practice.
EXAMPLE 3

• Nashua corporation (in NH) manufactures carbonless carbon paper.
• 1100 lft/min; used 3.6 lbs per 3000 sq ft.
• The operator was adjusting constantly and 3.6 lbs was high. Idea to buy a costly coating head.
• Statistics was used to determine that adjustments were made based on delayed
EXAMPLE 3 (cont’d)

- data and so it didn’t pertain to current conditions. New operating instructions led to fewer adjustments and reduced the average to 2.6lbs of dry coating/3000 sq.ft

- Resulted in a savings of $800,000/year in chemicals
EXAMPLE 4

- This deals with efficacy and tolerability of OMEP (omeprazole 20) vs RAN/MET (ranitidine/metoclopramide) in SEE (severe erosive esophagitis-Stomach acid in esophagus)

- Number and percentage of patients healed at 4 and 8 weeks.

- Median time to relief (days) of SEE
Healing rate and MTR

- 4-week
- 8-week
- Daytime
- Nighttime

OMEP
RAN/MET
EXAMPLE  5

- Health insurance: Should there be a free medical care or how much deductible and co-payment one should pay?

- In 1974, the federal government set up a large experiment and used statistics to see how the premium and the usage of the facilities are related.
Care use vs Insurance

visits
admission
spending

0 25
50 95
Analysis and Influence

- Comparing persons paying 95% of the bills and persons paying nothing, we see
  - 40% fewer doctor visits
  - 23% reduction in hospital usage
  - 31% reduction in total spending
- Between 1982 & 1984, there was an increase in the amount of cost-sharing in private health insurance in USA
Change between 1982 & 1984

A = % of private insurance with a ded. for hospital visits
B = % of private insurance with $200 or more in ded.
C = Hospital discharges per 100; D = Doctor visits