Kinawa

Mathematical Circle

K-ELEMENT LISTS, FACTORIALS, PERMUTATIONS

The number of k-words. *If an alphabet contains N letters, then the number of all words (those that make sense, and those that don't) of k different letters is equal to* $(N)_k = N(N-1)...(N-k+1)$ *(There is a total of k factors). For example, using the alphabet of three letters A,B,C we can make 6 two-letter words: AB, AC, BA,BC, CA, CB).*

1. **a)** How many three-digit numbers are there that satisfy the following condition: all their digits are different and non-zero? (e.g. 987 is good but 988 is not)
   b) How many three-digit numbers are there that satisfy the following condition: no more than two of their digits are distinct and all digits are non-zero? (e.g. 988 is good but 987 is not)

2. How many permutations of 3 **different** digits are there, chosen from the ten digits 0 to 9 inclusive?

3. How many permutations of 4 **different** letters are there, chosen from the twenty six letters of the alphabet?

4. A committee consists of a Chairman, Vice Chairman, Secretary, Undersecretary, and an Ordinary Member. In how many ways can such a committee of 5 be chosen from 10 people?

5. Jones is the Chairman of a committee. In how many ways can a committee of 5 be chosen from 10 people given that Jones must be one of them?

6. A password consists of four **different** letters of the alphabet. How many different possible passwords are there?

7. A password consists of two letters of the alphabet followed by three digits chosen from 0 to 9. Repeats are allowed. How many different possible passwords are there?

8. **a)** There are eight horses taking place in the race. People are filling out the forms predicting what horse takes what place to make bets. Anyone who guesses the first four places wins ONE million. How many forms should Peter fill out to be sure to win?
b) A groom, Peter's friend, knows for sure which four horses will come first and in what order, however he is not allowed to tell anyone. Peter may try to guess and the groom may give him a wink if Peter is right. Peter can make complicated statements like "The horse A will outrun exactly one of the horses B,C,D" What is the minimal number of statements Peter needs to make to fill out the form and win?

9. **a)** There are two counterfeit coins among 6. One is heavier than the genuine, the second one is lighter. Is it possible to find both counterfeit coins and determine which is heavier and which is lighter using only three weighings on the balance scale without extra weights?
b) Among 4 coins there are two counterfeit – one is heavier and the other is lighter. What is the minimal number of weighings on the balance scale without weights one must make to find out for sure both counterfeit coins and to determine which is lighter?

Theory. *N objects can be put in a ordered sequence in* $N!=N(N-1)\ldots(2)(1)$ *ways (**N-factorial**). For instance,* $4!=(4)(3)(2)(1)=24$.

10. Compute 1!, 2!, 3!, 5!, 6! and 7!

11. An encyclopedia has eight volumes. In how many ways can the eight volumes be placed on the shelf?

12. Assuming that any arrangement of letters forms a 'word', how many 'words' of any length can be formed from the letters of the word SQUARE?

13. 16 teams enter a competition. They are divided up into four Pools (A, B, C and D) of four teams each.

Every team plays one match against the other teams in its Pool. After the Pool matches are completed:
• the winner of Pool A plays the second placed team of Pool B
• the winner of Pool B plays the second placed team of Pool A
• the winner of Pool C plays the second placed team of Pool D
• the winner of Pool D plays the second placed team of Pool C
The winners of these four matches then play semi-finals, and the winners of the semi-finals play in the final.
How many matches are played altogether?

14. A restaurant offers 5 choices of appetizer, 10 choices of main meal and 4 choices of dessert. A customer can choose to eat just one course, or two different courses, or all three courses. Assuming all choices are available, how many different possible meals does the restaurant offer?

15. How many four-digit numbers are there such that the number contains all digits greater than 5 and every digit occurs exactly once?

16. Find two last digits of 11! without computing the number itself.

17. How many ways are there to put 8 rooks on the chessboard in such way that they can not capture each other?

18. Compute **a)** 2011!/2010! **b)** 100!/98!

19. There are four stones of different weights and a balance scale without weights. What is the minimal number of weighings required to place the stones in order from the lightest to the heaviest?

20. What number is larger 100! or 2^{100}? Why?

21. Seven cockroaches participate in a roach race. For a price of $200 you can fill a form with prediction what roach takes what place. If you guess you win One million dollars. Is it reasonable to participate if you have no information on these roaches?

TAKE A CHALLENGE!!!

22* a) There are four towns in a country. Every two towns are connected by a road. A dark wizard wants to make all roads one-way such that if there is a way from town A to town B, then there is no way from B to A (even through the other towns). How many ways are there to throw such spells on the roads?

b) The same question about ten towns.

23* There are four weights. It is known that their masses are 101, 102, 103, and 104 grams; however it is unknown how much each one weighs. What is the minimal number of weighings required to determine the mass of each weight?

24* Among five coins there are two counterfeit. The genuine weighs 10 grams each, one counterfeit weighs 9 grams, the other one weighs 11 grams. What is the minimal number of weighings required to find both counterfeit coins and to determine their weights?