Flow Through a Nozzle (example: Rocket Engine)

Consider an ideal gas flowing through a conical duct (nozzle) with variable cross sectional area wherein the duct surface is heated so that the gas temperature varies as it flows through the duct. In this case, the Mach number y varies through the duct.

The formula below was found in a textbook on numerical methods written by an engineer. It's supposed to be the differential equation that governs how the Mach number y changes with position x through the duct (nozzle).

$$\frac{dy}{dx} = \frac{y}{1-y^2}\left(1+\frac{\gamma-1}{2}y^2\right)\left[\frac{\beta}{2}\cdot\left(\frac{1+\gamma y^2}{cT_i+\beta x}\right)-\frac{2\alpha}{d_i+\alpha x}\right] \tag{1}$$

The parameters are:

- c is the specific heat at constant pressure, $\left[\dfrac{L^2}{T^2\Theta}\right]$
- γ is the ratio of specific heats for the gas, [dimensionless]
- β is the temperature gradient (the rate at which the temperature changes along the duct wall), $[\Theta/L]$
- d_i is the diameter of the duct inlet,
- T_i is stagnation temperature at the duct inlet,
- α is the grade (slope) of the duct. $\alpha<0$ for a constricting duct and $\alpha>0$ for an expanding duct, [dimensionless]
- x is the distance from the nozzle entrance,
- y_i is the Mach number at the duct inlet, [dimensionless]
- $y = y(x)$ is the Mach number at location x, [dimensionless]. The flow is subsonic at x if $y(x)<1$; the flow is supersonic at x if $y(x)>1$.

Problem: Determine whether Eqn. (1) is dimensionally consistent. If not, explain clearly what is wrong.